ROBOTICS SCIENCE AND SYSTEMS 2021

Model-based Design and Control of Dynamic Legged Robots

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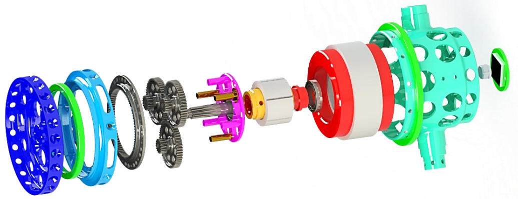
Hubo Lab
Humanoid Robot Research Center

KAIST Department of Mechanical Engineering

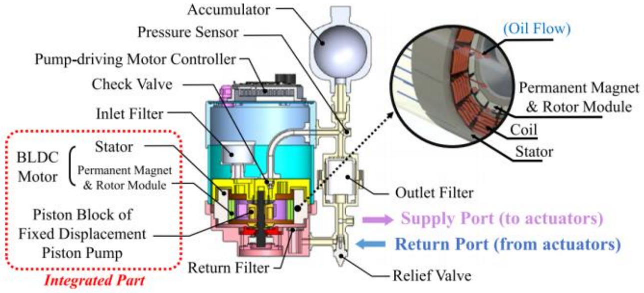
DRCD Lab: Dynamic Robot Control and Design Laboratory

Research on Design, Control, State Estimation of Legged Robot Systems

Actuator Design

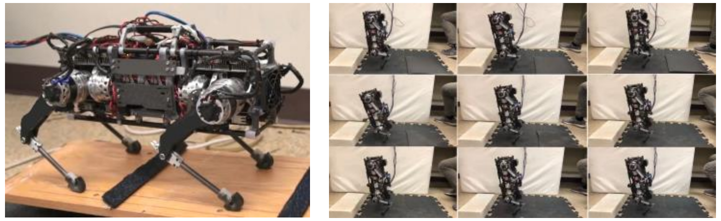


Quasi Direct Drive Design [IROS'17]
(IROS Best Student Paper Finalist)

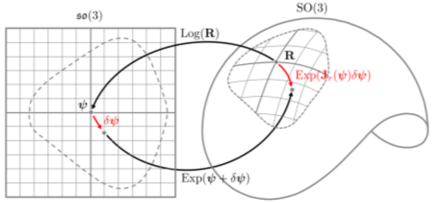


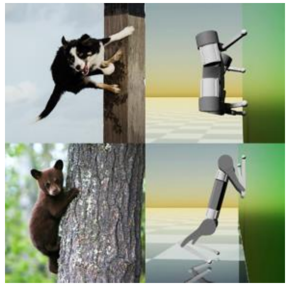
Hydraulic Power Unit Design [RA-L'21]

Quadrupedal Robots



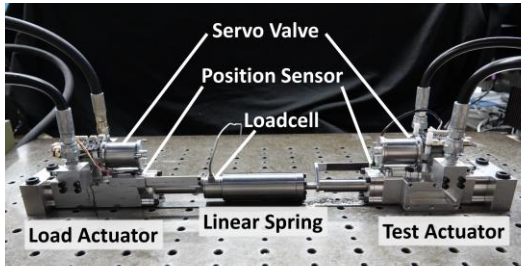
Representation-free MPC [T-RO'21]
('20 TC Best Paper Finalist)



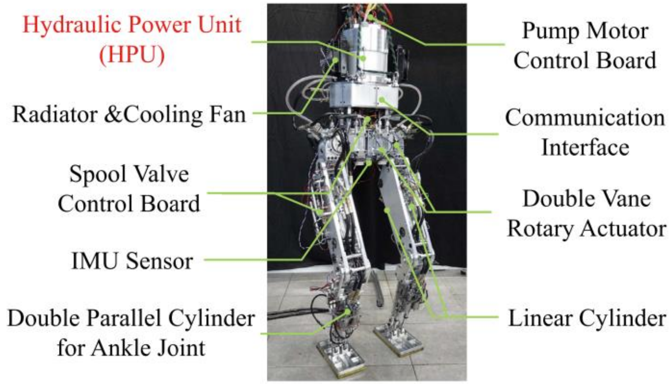


Nonlinear MPC on SO(3) [IROS'20]
(IROS Best RoboCup Paper)

Humanoid Robots



Learning-based Force Control [RA-L'21]



Hydraulic Humanoid [RA-L'21]

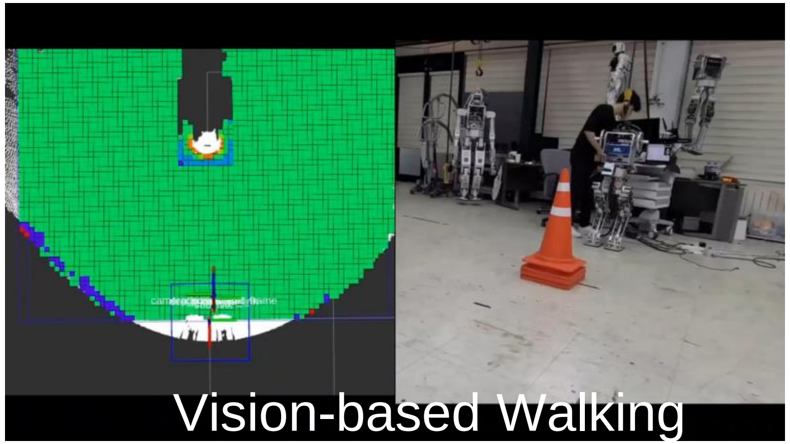
Research Work in the DRCD Lab

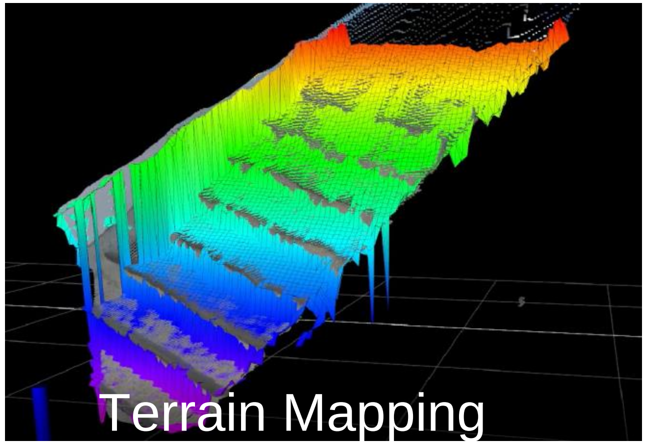

Hydraulic Quadruped

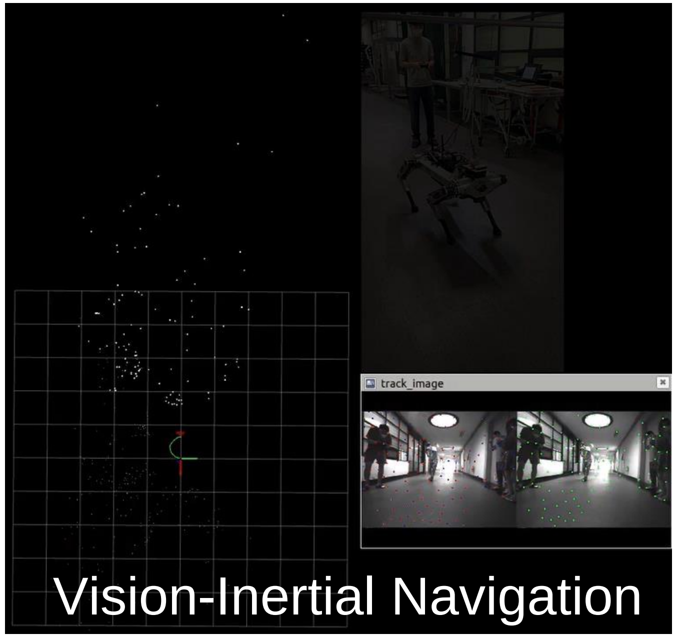

Hydraulic Biped


Dynamic Qaudrupeds

Collaboration with Other Labs


Vision-based Walking


Terrain Mapping



Vision-Inertial Navigation

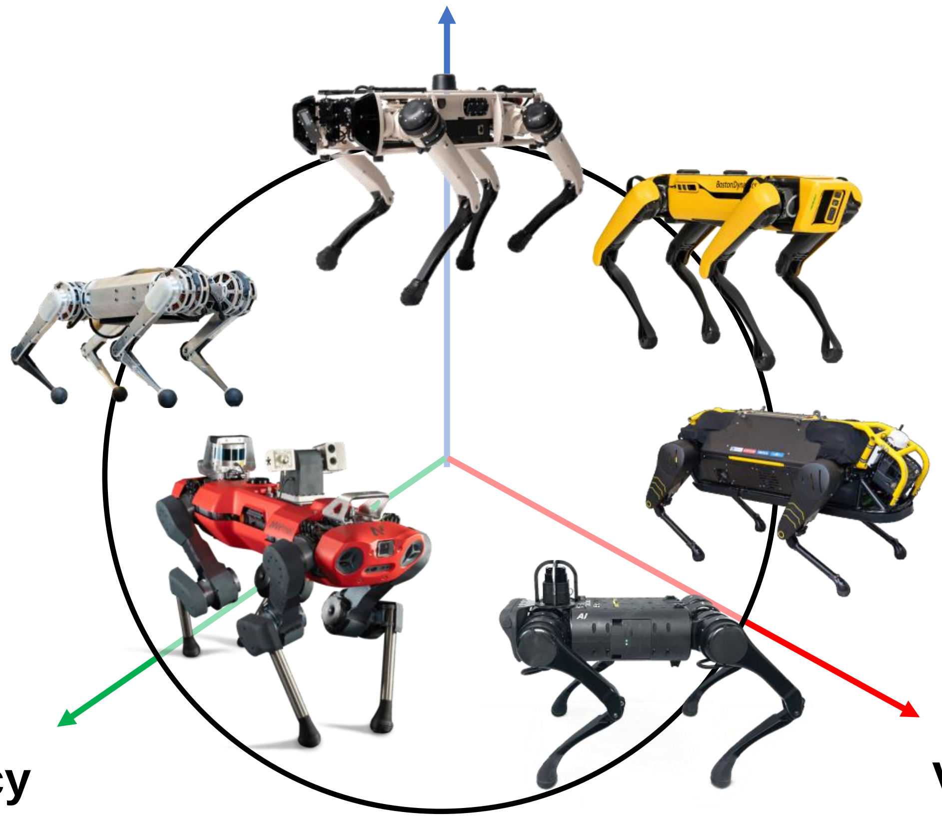
Great Examples of Legged Systems in Biology

Athletic Mobility in Complex Environments
"Super squirrel" from National Geographic

Stability with Body Coordination
Rock Climbing without Hands (gfycat.com)

Dynamic Balance while Fast Leg Kicking
Kazotsky Kick of Ukrainian Dance Company (youtube.com)

Three Virtues of a Great Legged Robot System

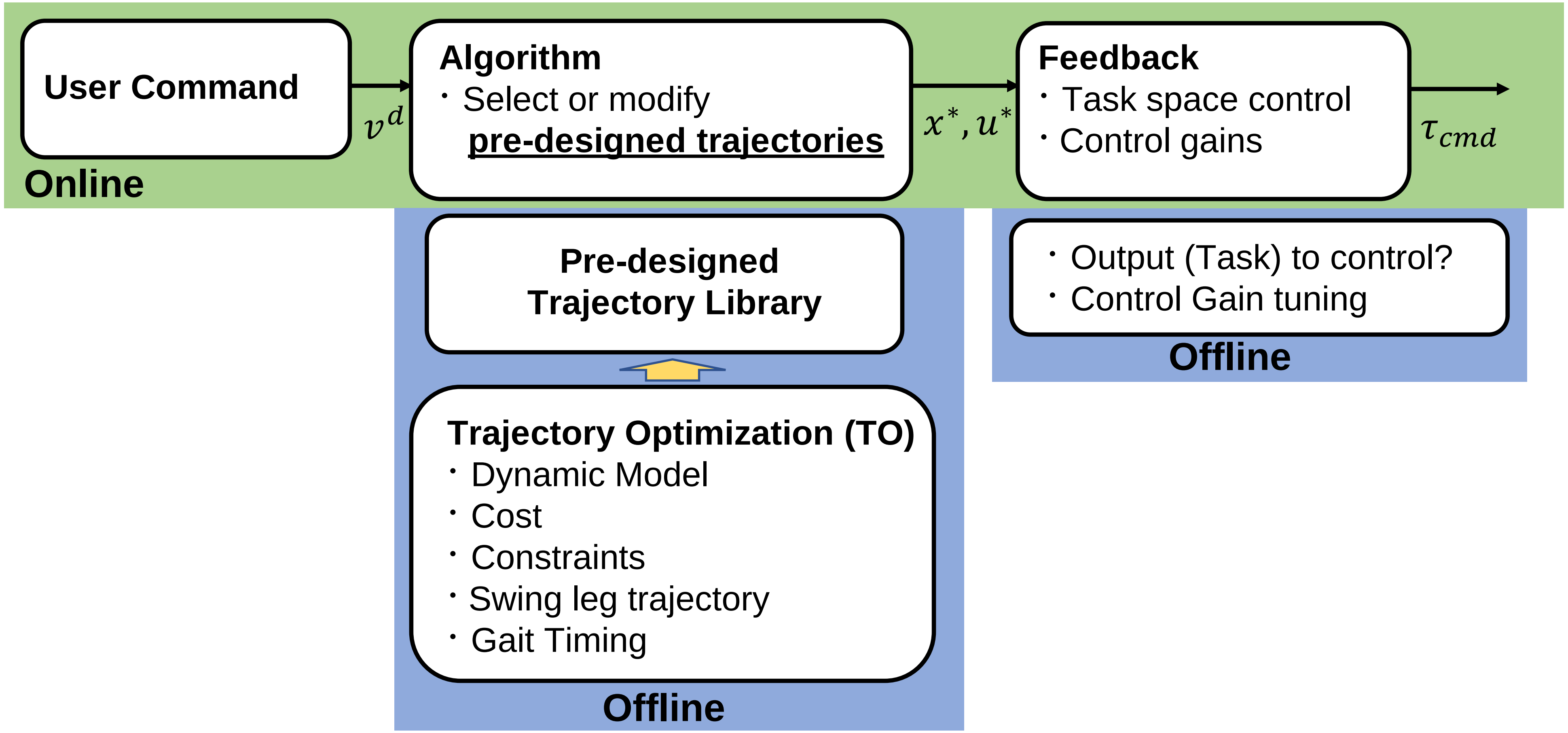


Control Algorithms
- Exploit diverse model structures
- Responsive to the environment
- Ability to control a variety of maneuvers
- Real-time computation

Actuator Design
- High torque and high speed
- Transparent transmission
- Fast response to the commanded torque
- Low inertia and friction

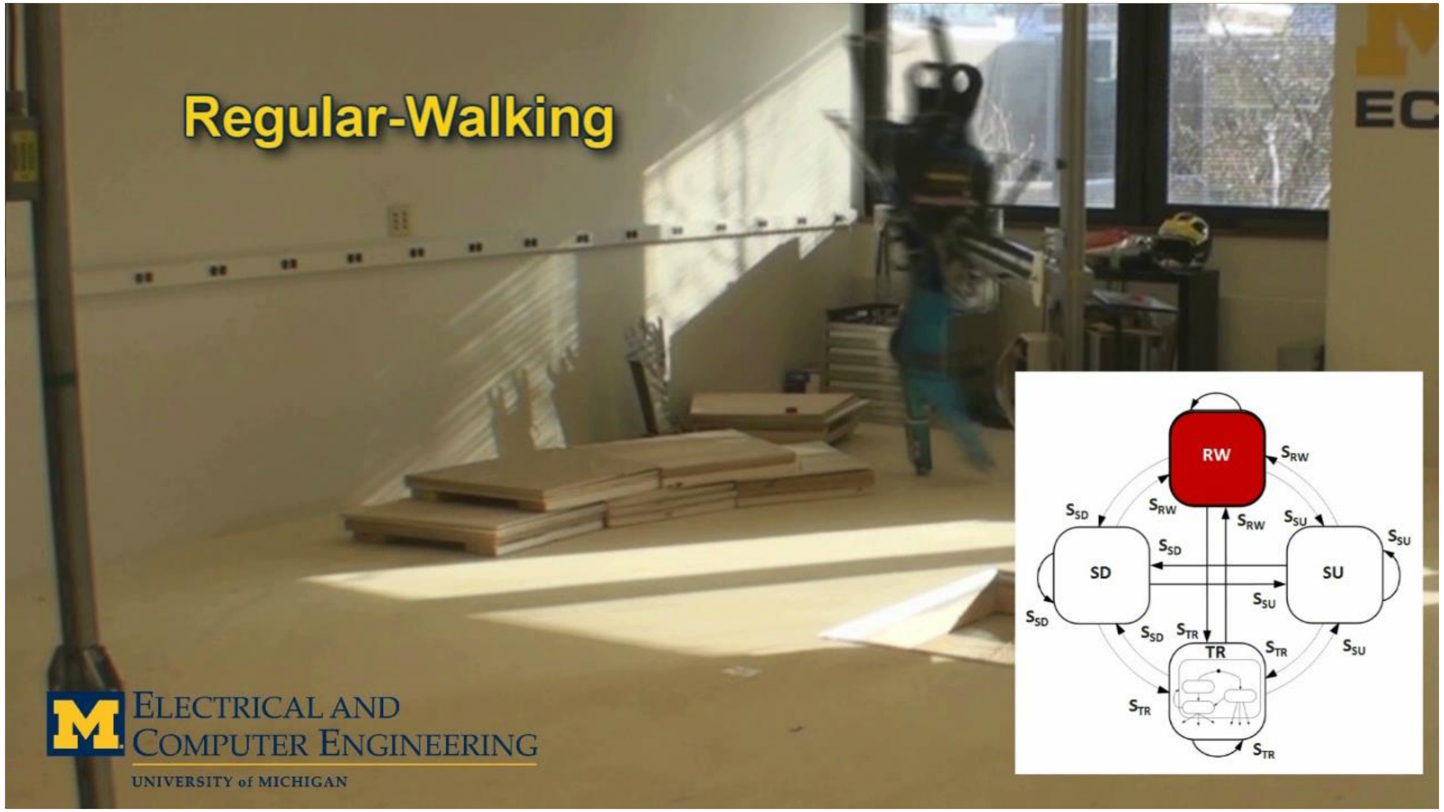
Model-based optimization

Conventional Control Design for Legged Robots



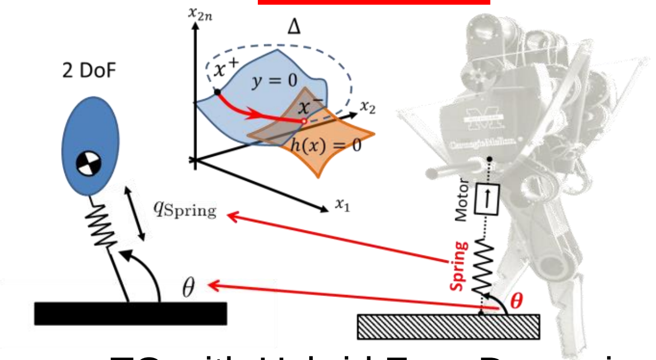
Using Trajectory Library [T-RO'12, ICRA'12, IJRR'11],

- Trajectories for various types of obstacles are generated by **offline optimization** (hybrid zero dynamics)
- A **heuristic design** of finite-state-machine is introduced to manage switching between trajectories

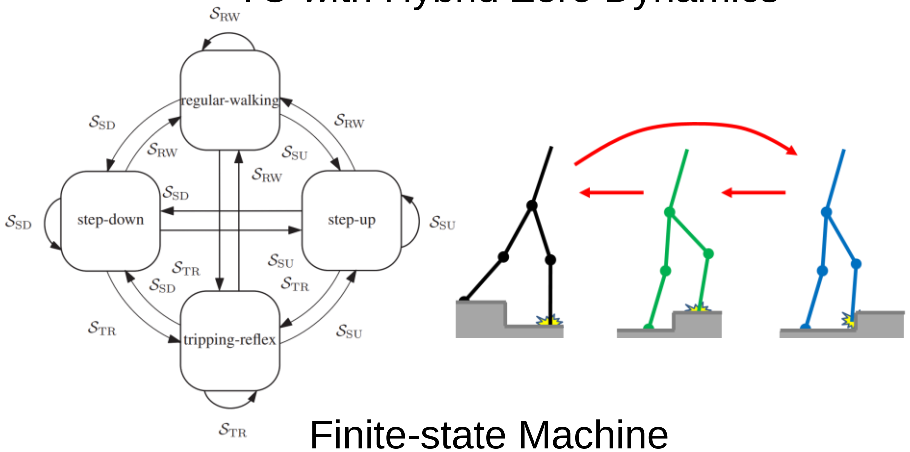


$$y = h_0(q) - h_d(\theta(q)) \equiv 0$$

6 DoF



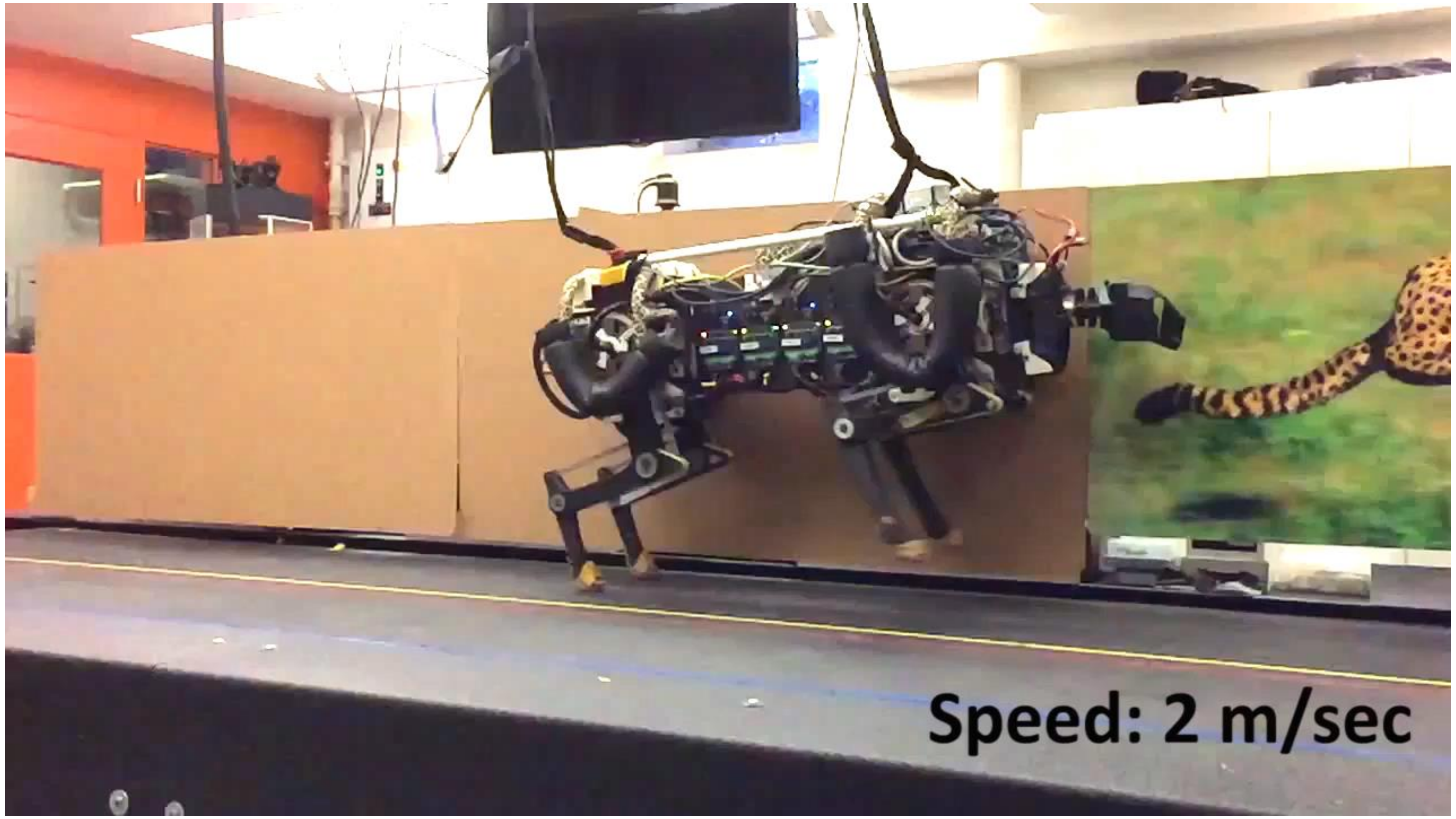
TO with Hybrid Zero Dynamics

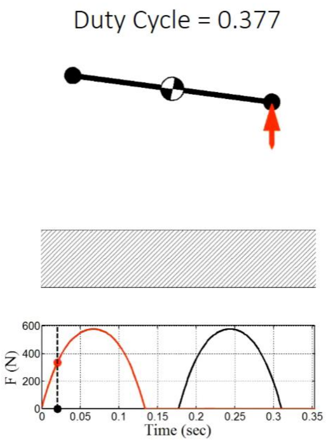


Finite-state Machine

Modifying Pre-Obtained Trajectory [IJRR'17, ICRA'15, IROS'14]

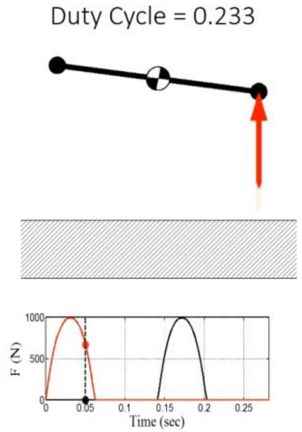
- Periodic trajectory for a simplified model obtained from **off-line optimization**
- **Online modification** of trajectories using impulse-planning for different speeds.

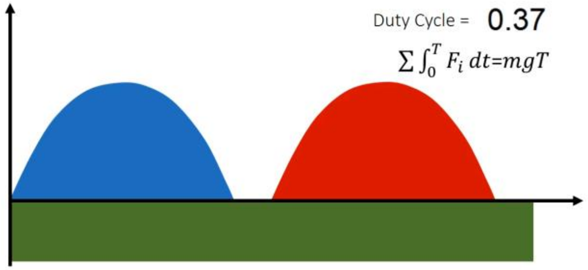




Momentum Balance

$$\int_0^T (F_z^* - mg)dt = 0$$





Online Optimization for Jumps over Obstacles [RSS'15, RAS'21]

On-line

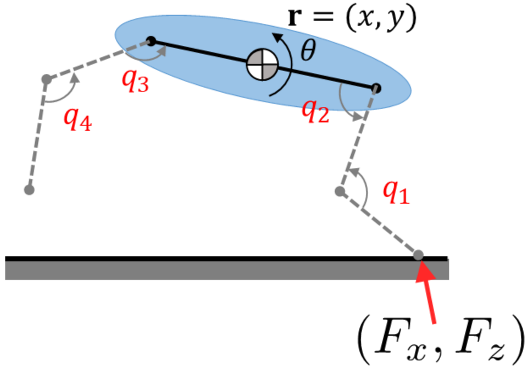
User Command → v^d →

Algorithm
- Obstacle Detection
- Switching to the online trajectory generation

→ x^*, u^* →

Feedback
- Task space control
- PD gains

→ τ_{cmd}

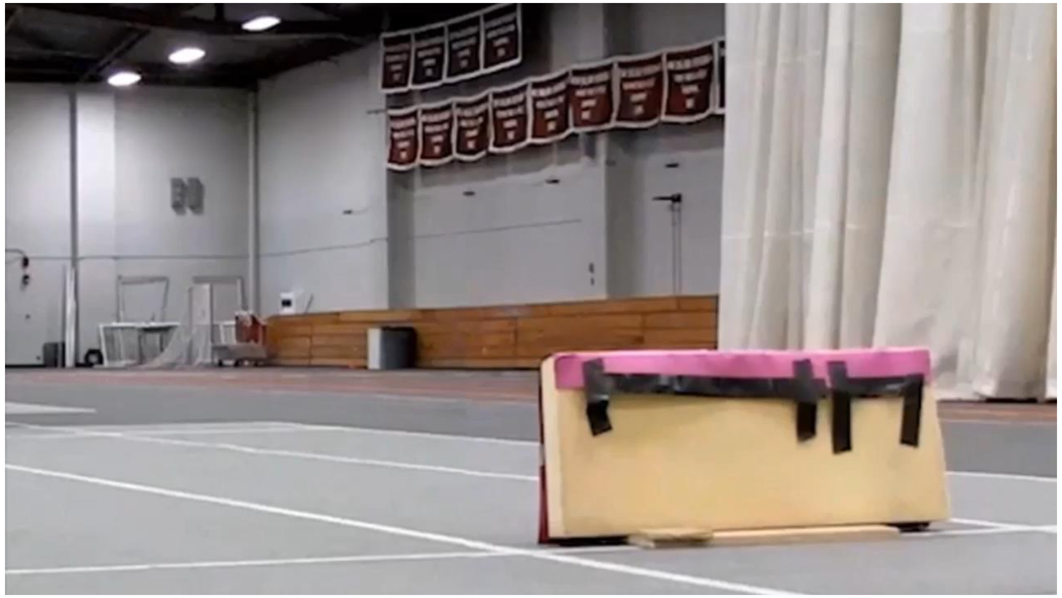


Trajectory Optimization
- Simplified Model
- Cost
- Constraints
- ~~Swing leg trajectory~~
- ~~Gait Timing~~

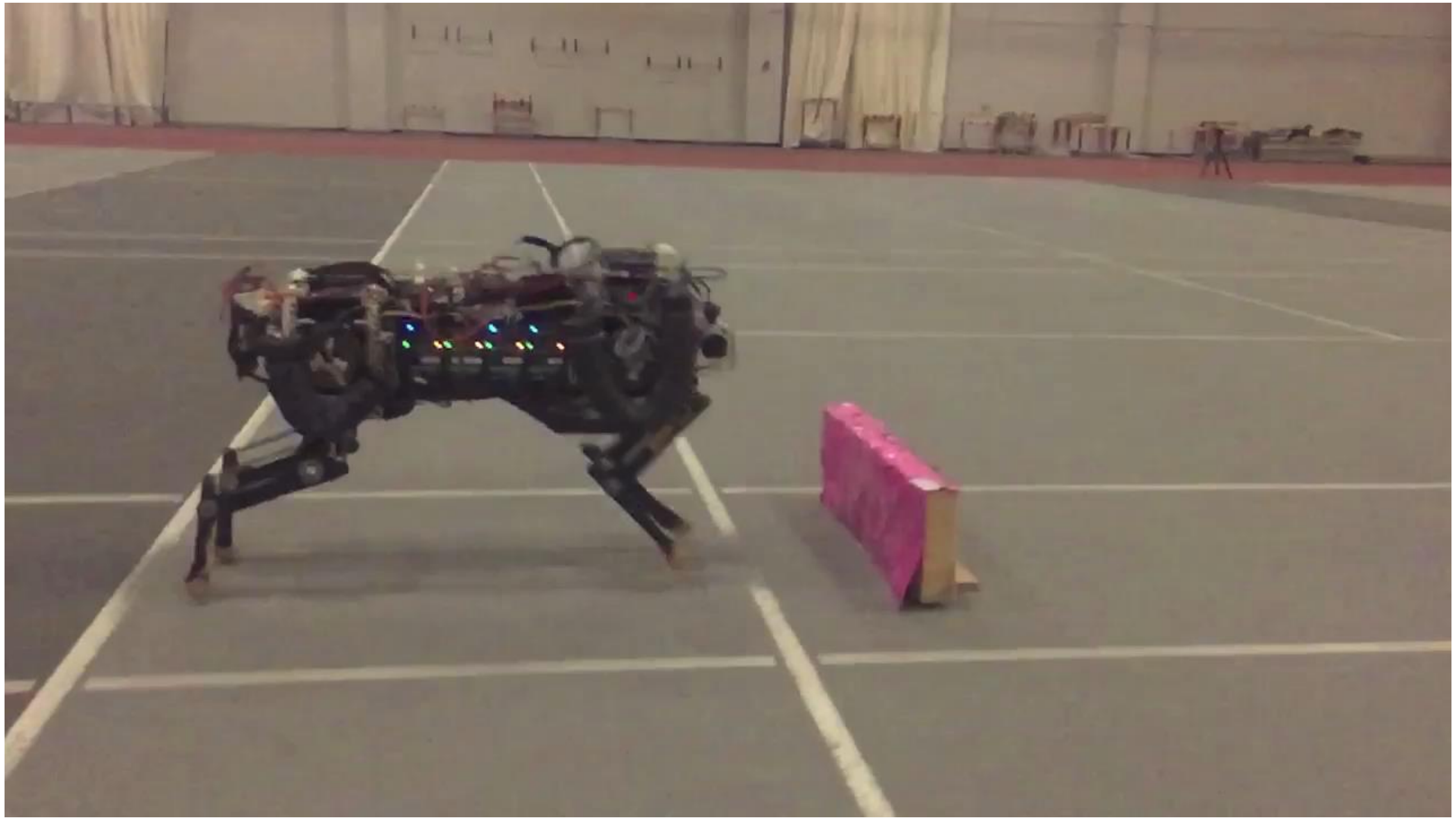
On-line

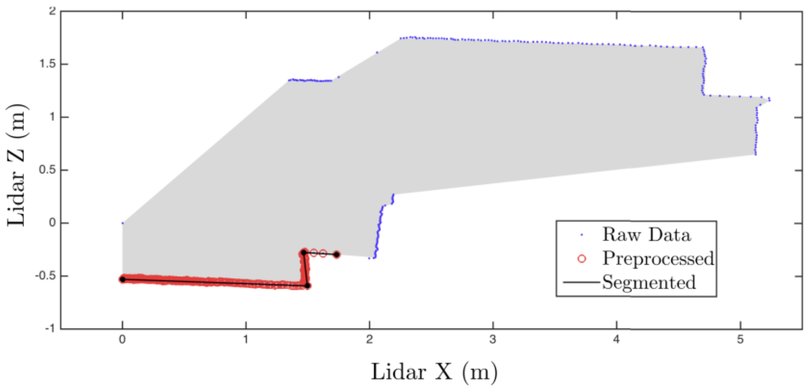
- Output (Task) to control?
- Control Gain tuning

Off-line

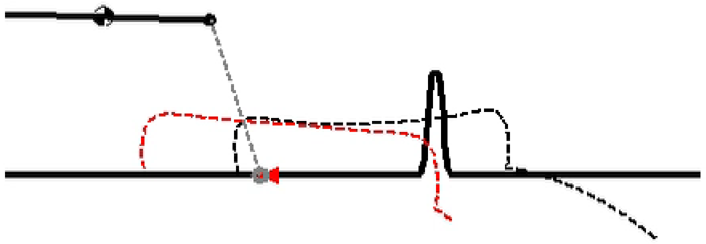


Online Optimization for Jumps over Obstacles [RSS'15, RAS'21]



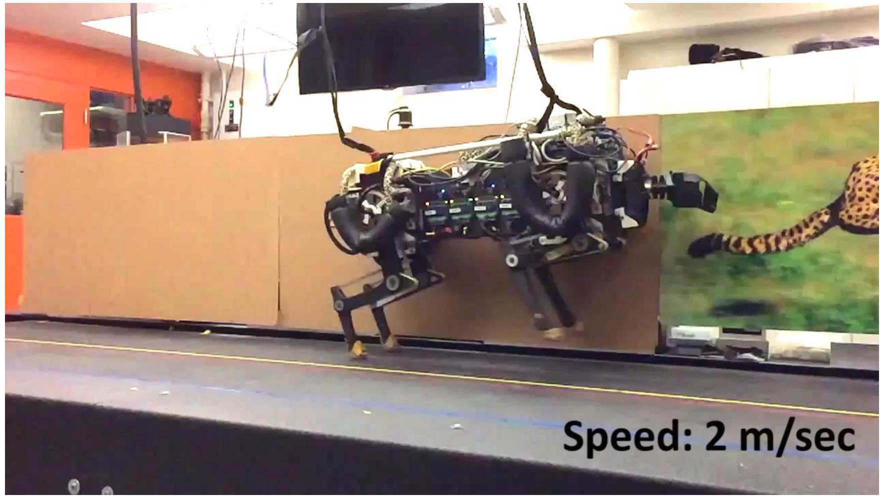


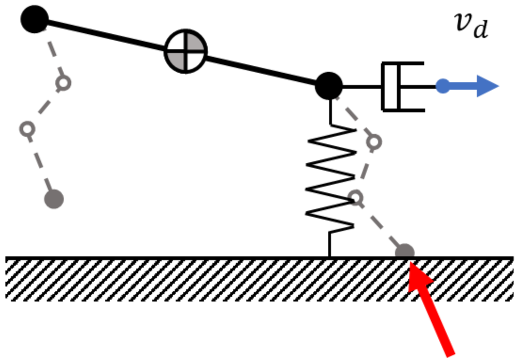
Obstacle Detection



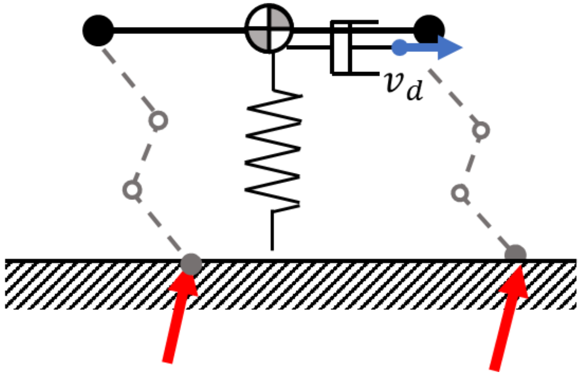
Online Jump TO <100 msec

Heuristic Output (Task) Choices in Control Design





Bounding



Trotting

Complex 3D Dynamic Motions

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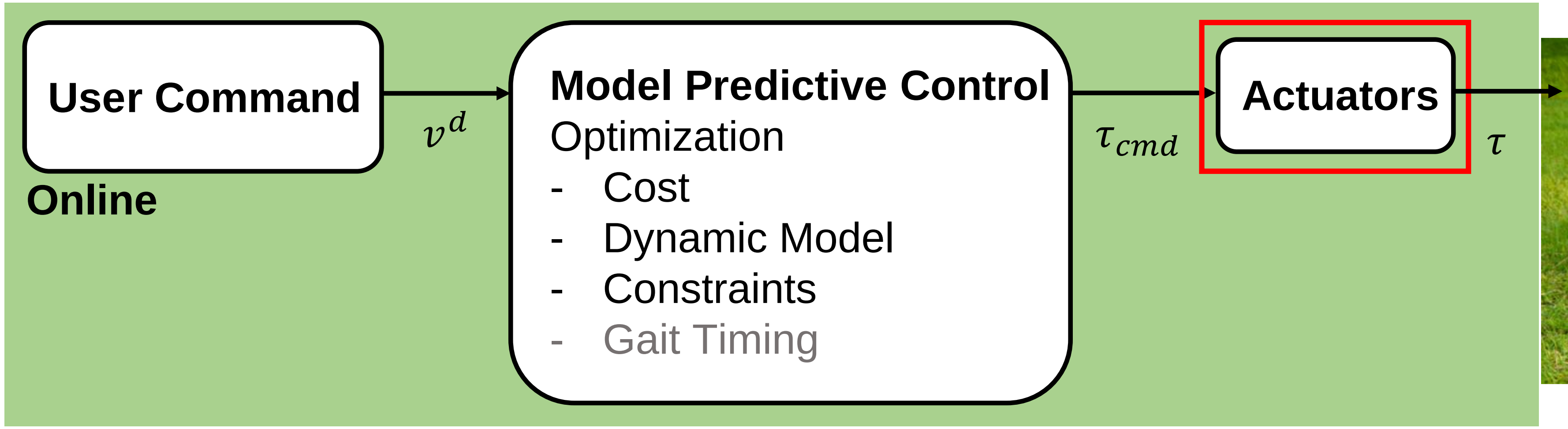
Mechanical Engineering

Model Predictive Control for Legged Robots



Finite Time Optimal Control Problem

minimize $\sum_{t}^{t+T} \text{Cost}$

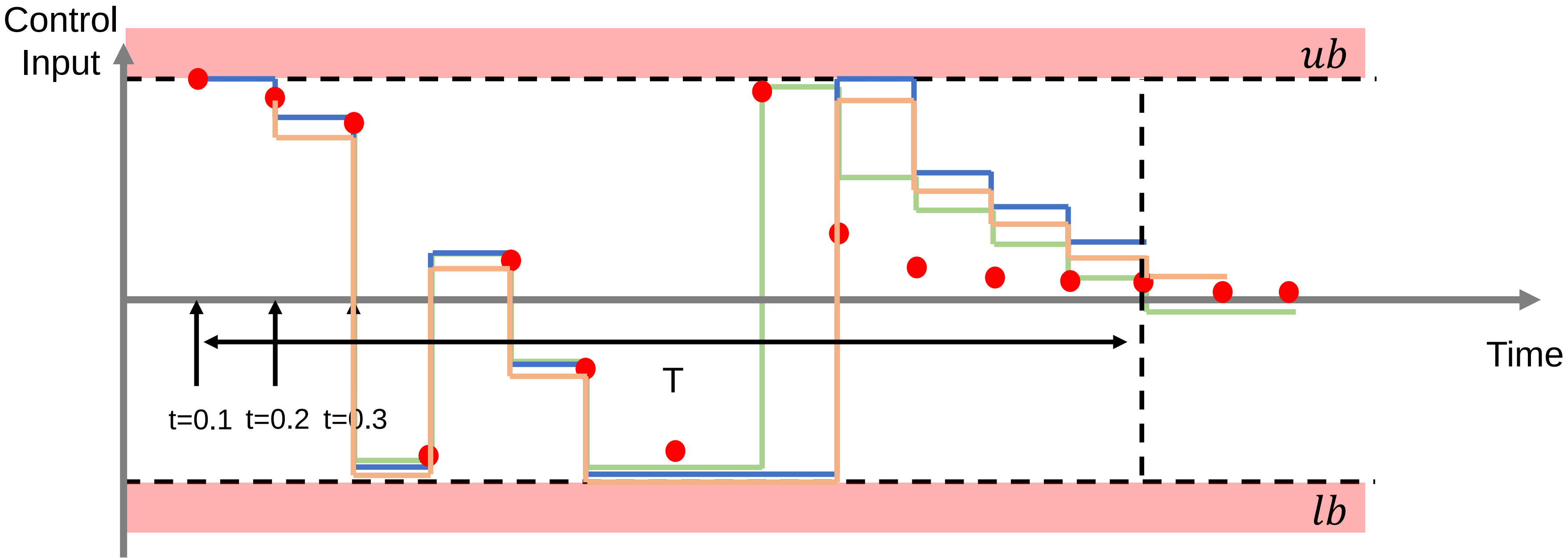
$x \to x_d, u \to u_d$

subject to Equation of Motion

Constraints

$lb \leq u \leq ub$

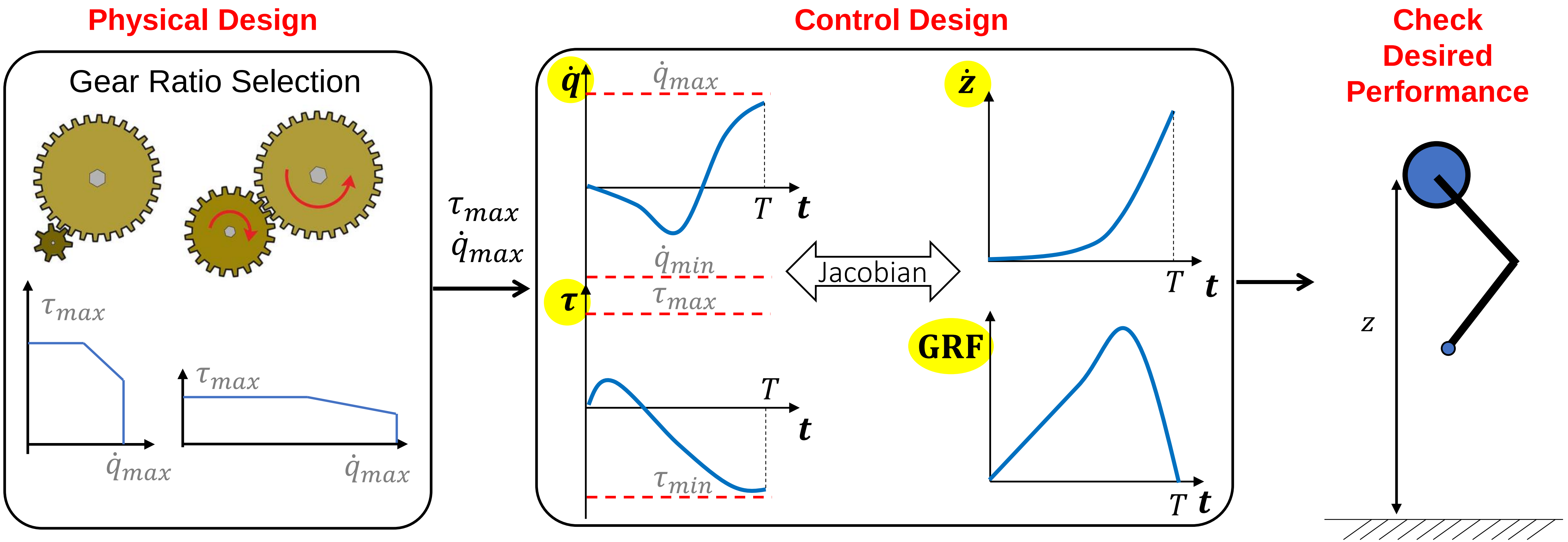
$x \in \mathcal{F}_x$



KAIST Department of Mechanical Engineering

Torque Control Actuator Design [IROS'17, Best Student Paper Finalist]

- Choose a right combination of gear ratio and motor choice
- Integrated approach for physical and control system design using nonlinear program



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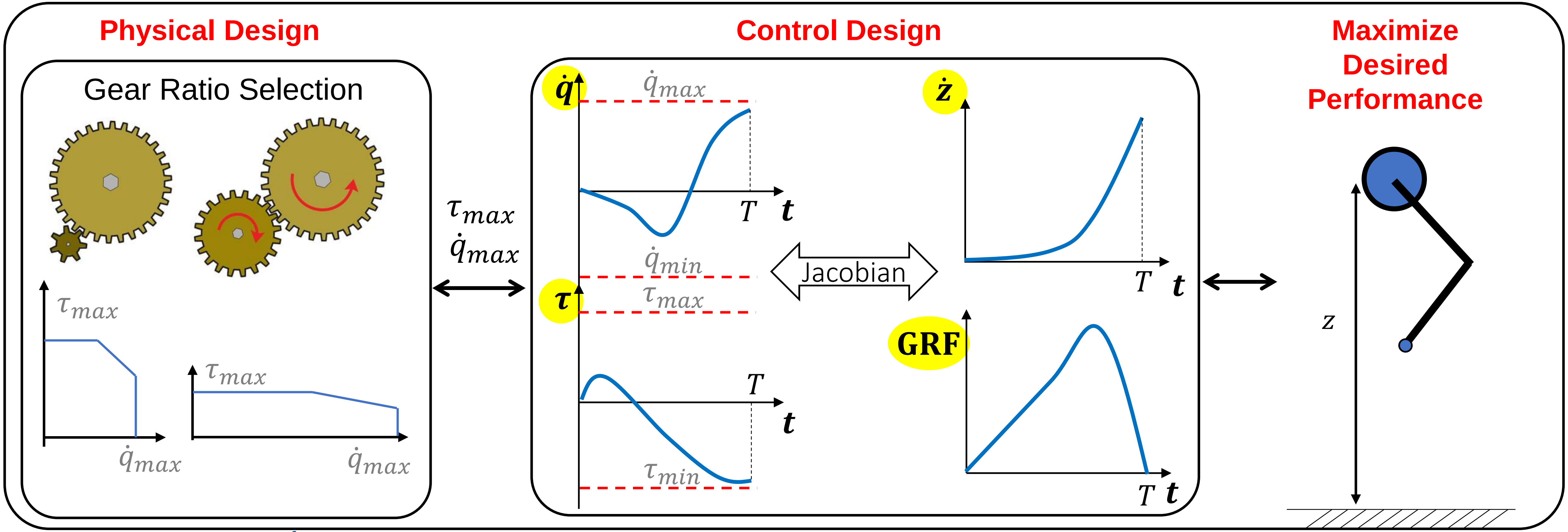
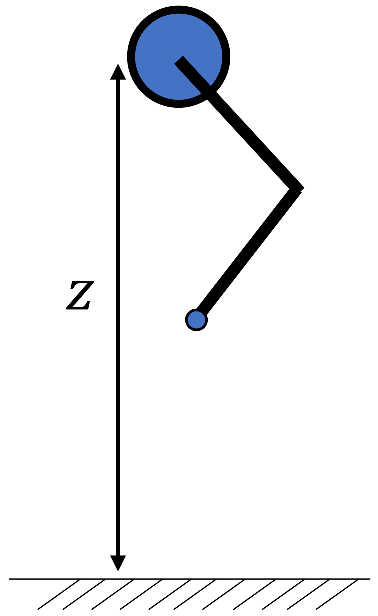
Nonlinear Optimization Problem



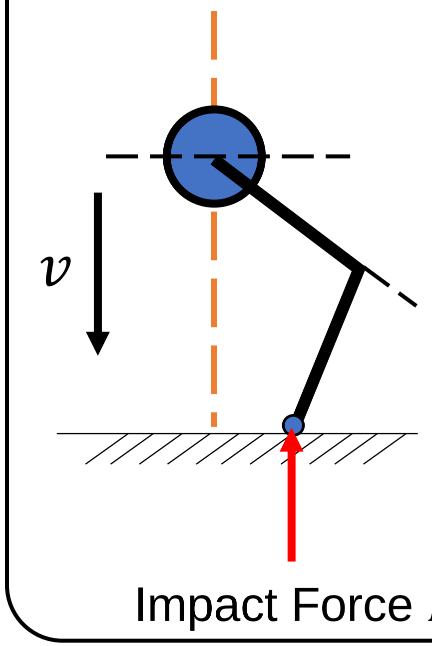
Torque Control Actuator Design [IROS'17, Best Student Paper Finalist]

- Select a gear ratio (and motor specs) to
  - Maximize dynamic maneuvering capability (jumping height)
  - Consider impact force when landing
  - While respecting motor speed and torque limitations

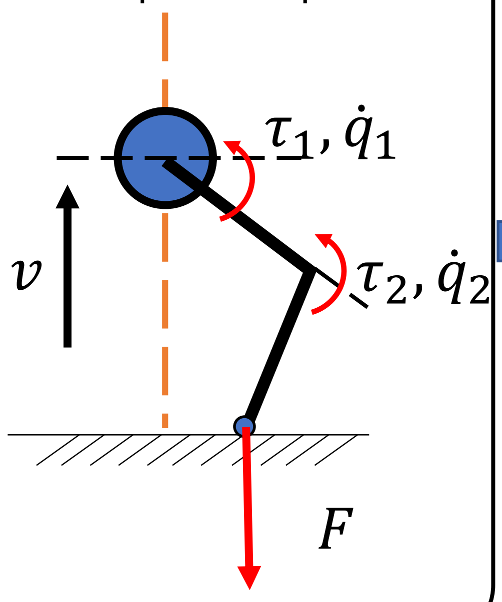
Maximize Jumping Height



Impact Force upon Landing



Limitation on actuator torque and speed



Nonlinear Programming

Optimization Variables

Gear ratio, GRF profile, Impact Force $q, \dot{q}$

Objective

Maximize Jumping height
Reduce Impact force

Constraints

Dynamics

Impact Model

Forward Kinematics

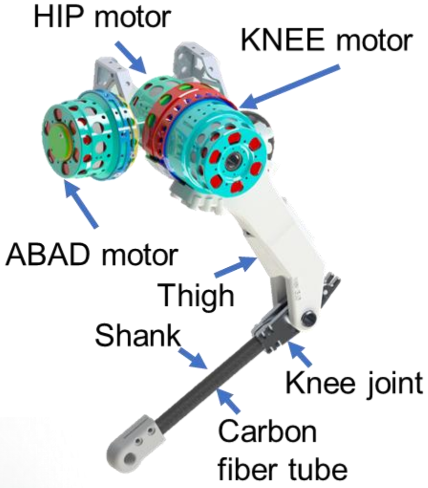
Motor Speed and Torque

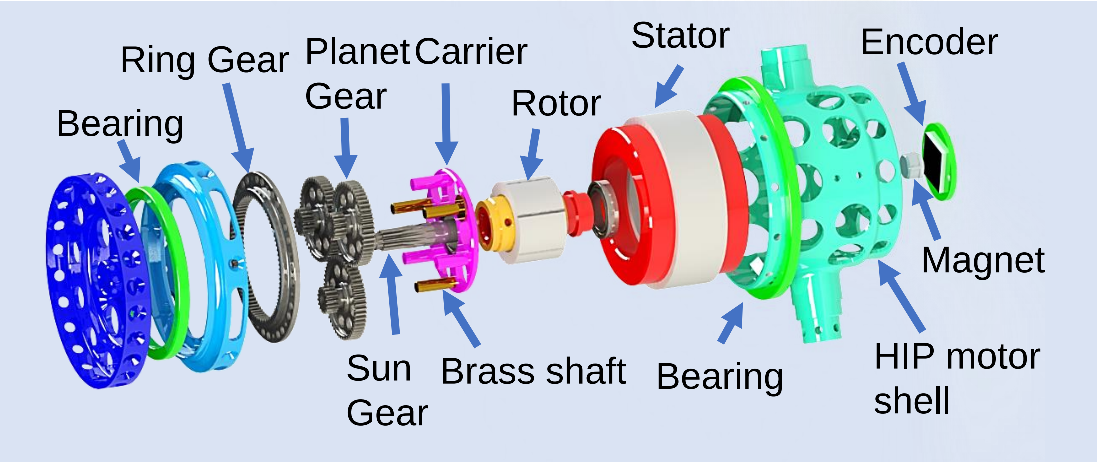
Gear ratio $= 22.9:1$

Hardware Integration

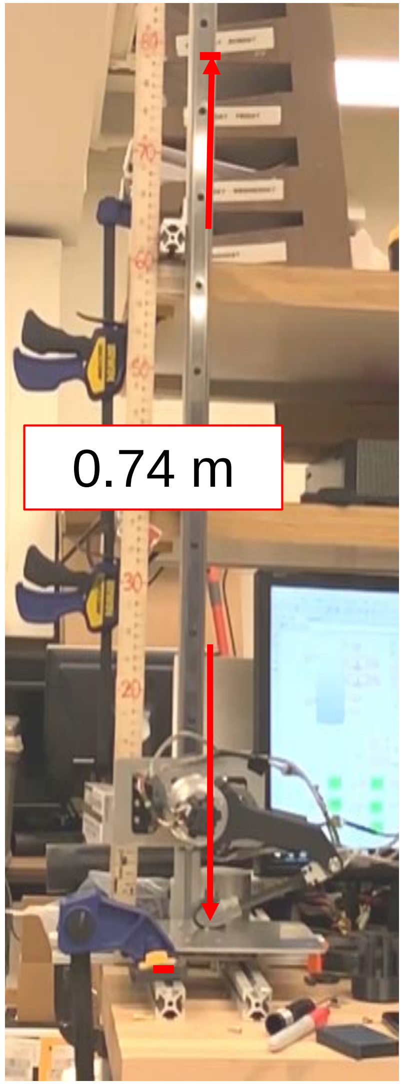
Leg module specifications:

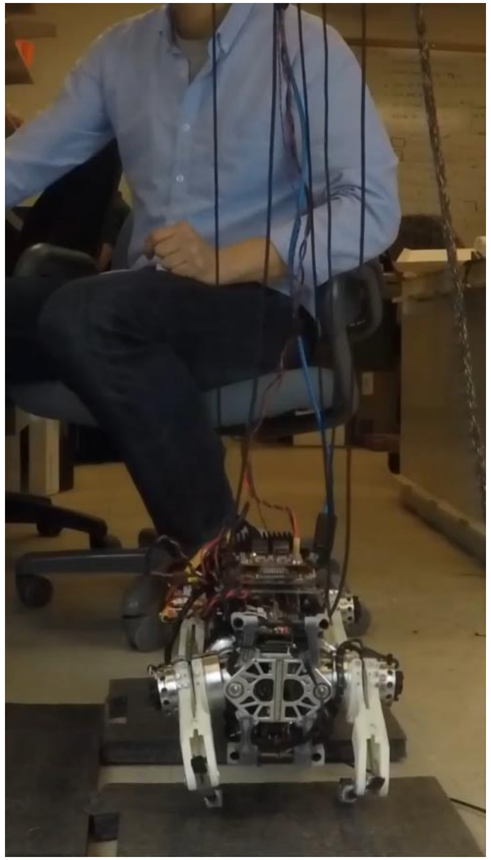
- Composed of 3 motor modules
- Total mass: 0.89 kg
- Link length $l = 0.14$ m
- Total link weight 0.06 kg (<10%)





Compound Planetary Gear





Optimal Control Problem

Cost: $c_f(\boldsymbol{x}_\mathbf{T}) + \int_0^T (c_x(\boldsymbol{x}, \boldsymbol{x}_\mathbf{d}) + c_u(\boldsymbol{u}, \boldsymbol{u}_\mathbf{d}))dt$

Dynamics: $m\ddot{\mathbf{c}} = \sum_j^N \boldsymbol{F}_\mathrm{j} + m\boldsymbol{g}$

$$\ddot{\boldsymbol{\varphi}} = \boldsymbol{f}(\boldsymbol{\varphi}, \dot{\boldsymbol{\varphi}}, \boldsymbol{F}_j)$$

Parameterization of $\mathbf{R}(\boldsymbol{\varphi})$

where, $\boldsymbol{\varphi} \in \boldsymbol{R}^{3X1}$

Constraints: $\boldsymbol{F}_j \in \mathcal{F}$

Discretization
Linearization

Optimal Control Problem for Discrete Linear System

$$\min \sum_i^{N_T} \boldsymbol{x}_i^T \mathbf{Q} \boldsymbol{x}_i + \boldsymbol{u}_i \mathbf{R} \boldsymbol{u}_i$$

$$\boldsymbol{x}_{\mathrm{i+1}} = \mathrm{A_i}\boldsymbol{x}_\mathrm{i} + \mathrm{B_i}\boldsymbol{u}_\mathrm{i}$$

$$\boldsymbol{\Phi u}_i \leq \mathbf{h}$$

Transcription

Quadratic Programming

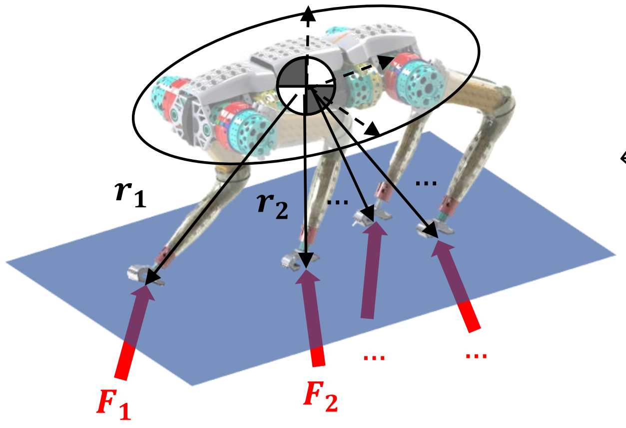
$$\min \frac{1}{2}\boldsymbol{x}^\mathbf{T}\mathbf{P}\boldsymbol{x} + \mathbf{g}^\mathbf{T}\boldsymbol{x}$$

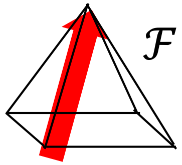
$$\mathbf{G}\boldsymbol{x} \leq \boldsymbol{h}$$

$$\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$$

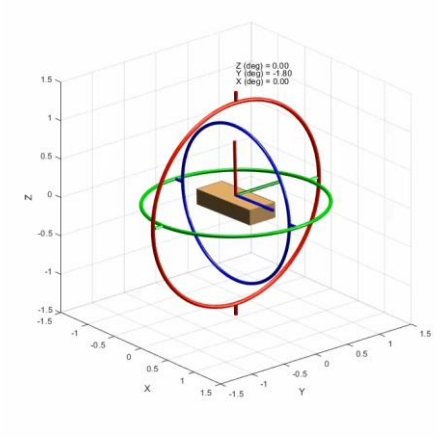
P: Sparse PSD
G, A: Sparse

$\boldsymbol{x} \in \mathbf{R}^{210X1}$
$\boldsymbol{h} \in \mathbf{R}^{112X1}$
$\mathbf{A} \in \mathbf{R}^{84X1}$

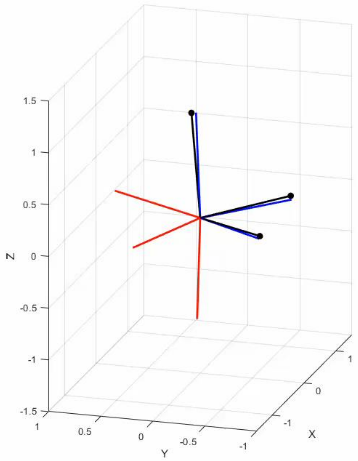




Euler Angles

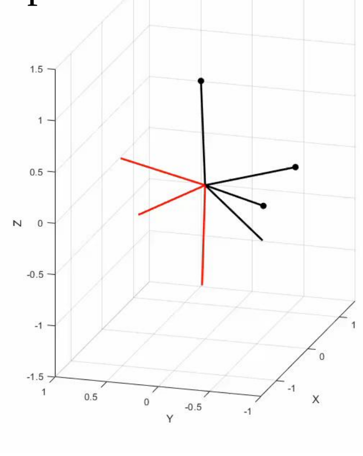


Gimbal Lock (Singularity)



Not the shortest path (of the rotation error)

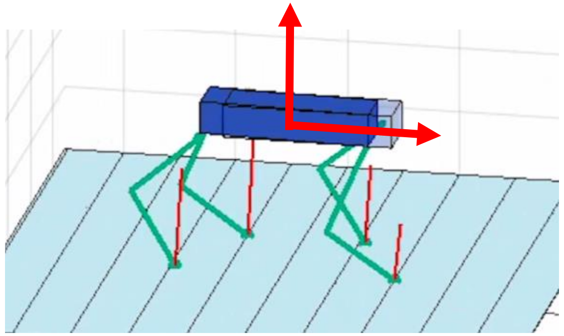
Exponential coordinates



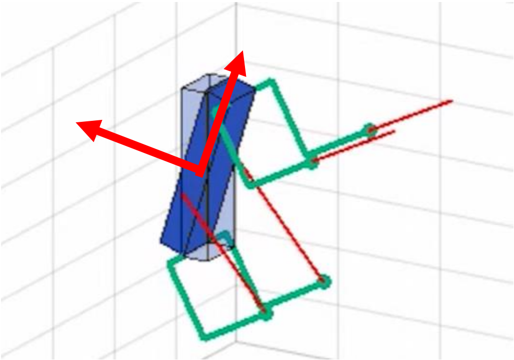
Shortest path (of the rotation error)

Linear Representation-free MPC on SO(3)

[ICRA'19, T-RO'21(TC Best Paper Finalist)]



Nominal Body Pose

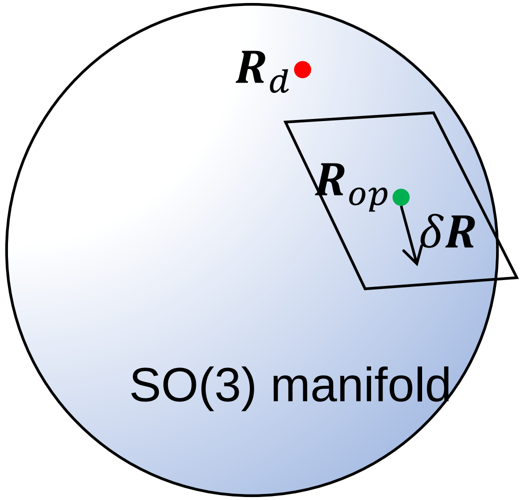


Large angular excursion

From Youtube, Alex & Jumpy - The Parkour Dog

Linear Model Predictive Control on SO(3)

[ICRA'19, T-RO'21(TC Best Paper Finalist)]



Dynamics

Linear Rotational Dynamics

$$\mathrm{vec}(\dot{\boldsymbol{R}}_k) = C_\xi^c + C_\xi^\xi \boldsymbol{\xi}_k + C_\xi^\omega \boldsymbol{\omega}_k,$$

$$\boldsymbol{I}\dot{\boldsymbol{\omega}}_k = C_{\dot{\omega}} + C_{\dot{\omega}}^{\delta p}\boldsymbol{p}_k + C_{\dot{\omega}}^{\xi}\boldsymbol{\xi}_k + C_{\dot{\omega}}^{\omega}\boldsymbol{\omega}_k + C_{\dot{\omega}}^{\delta u}\delta\boldsymbol{u}_k$$

Cost

Approximate orientation error:

$$\boldsymbol{e}_R = \log\left(\boldsymbol{R}_d^{\mathsf{T}}\boldsymbol{R}_{op}\right)^{\vee} + \boldsymbol{\xi}_k$$

Cost of orientation error (p.d.)

$$\Psi = \boldsymbol{e}_R^T \boldsymbol{Q}_R \boldsymbol{e}_R$$

Linear MPC

$$\min_{U_0}. \sum_{k=0}^{N-1} \left|\boldsymbol{x}_k - \boldsymbol{x}_{d,k}\right|_{\boldsymbol{Q}_x}^2 + \left|\boldsymbol{u}_k - \boldsymbol{u}_{d,k}\right|_{\boldsymbol{R}_u}^2$$

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{B}\delta\boldsymbol{u}_k + \boldsymbol{d}_k$$

$$\boldsymbol{x}_k \in \mathcal{X}, \delta\boldsymbol{u}_k \in \mathcal{U}$$

$$\boldsymbol{x}_0 = \boldsymbol{x}(t)$$

$$\boldsymbol{Q} = \boldsymbol{Q}^T \succeq 0, \boldsymbol{R} = \boldsymbol{R}^T \succ 0$$

Variation-based Linearization [G Wu et al., IEEE Access'15]

$\hat{\boldsymbol{\xi}} = \delta\boldsymbol{R} \in \mathfrak{so}(3)$, where $\boldsymbol{\xi} \in \mathbb{R}^3$

Taylor expansion of the matrix exponential map

$$\boldsymbol{R}_k \approx \boldsymbol{R}_{op}\exp(\delta\boldsymbol{R}_k) \approx \boldsymbol{R}_{op}(\boldsymbol{1} + \delta\boldsymbol{R}_k)$$

The variation of angular velocity $\delta\boldsymbol{\omega}_k$

$$\boldsymbol{\omega}_k = \boldsymbol{\omega}_k - \boldsymbol{R}_k^T\boldsymbol{R}_{op}\boldsymbol{\omega}_{op}$$

Optimal Control Problem

Cost: $c_f(\boldsymbol{x}_\mathrm{T}) + \int_0^T (c_x(\boldsymbol{x}, \boldsymbol{x}_\mathrm{d}) + c_u(\boldsymbol{u}, \boldsymbol{u}_\mathrm{d}))dt$

Dynamics: $m\ddot{\boldsymbol{c}} = \sum_j^N \boldsymbol{F}_\mathrm{j} + m\boldsymbol{g}$

$$\dot{\mathbf{R}} = \mathbf{R}\widehat{\boldsymbol{\omega}}$$

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} = \mathbf{R}^T \sum_j^N (\boldsymbol{p}_j - \boldsymbol{c}) \times \boldsymbol{F}_j$$

Constraints: $\boldsymbol{F}_j \in \mathcal{F}$

Optimal Control Problem for Discrete Linear System

$$\min \sum_i^{N_T} \boldsymbol{x}_i^T \mathbf{Q} \boldsymbol{x}_i + \boldsymbol{u}_i \mathbf{R} \boldsymbol{u}_i$$

$$\boldsymbol{x}_{\mathrm{i+1}} = \mathbf{A}_\mathrm{i}\boldsymbol{x}_\mathrm{i} + \mathbf{B}_\mathrm{i}\boldsymbol{u}_\mathrm{i}$$

$$\boldsymbol{\Phi}\boldsymbol{u}_i \leq \mathbf{h}$$

Quadratic Programming

$$\min \frac{1}{2}\boldsymbol{x}^\mathrm{T}\mathbf{P}\boldsymbol{x} + \mathbf{g}^\mathrm{T}\boldsymbol{x}$$

$$\mathbf{G}\boldsymbol{x} \leq \boldsymbol{h}$$

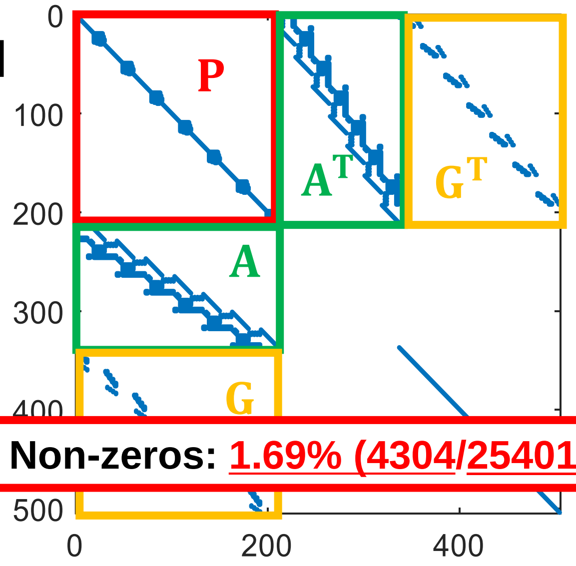
$$\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$$

$\mathbf{P}$: Sparse PSD
$\mathbf{G}, \mathbf{A}$: Sparse

$\boldsymbol{x} \in \mathbf{R}^{210X1}$
$\boldsymbol{h} \in \mathbf{R}^{112X1}$
$\mathbf{A} \in \mathbf{R}^{84X1}$

Sparse QP solver for MPC [RA-L'19]

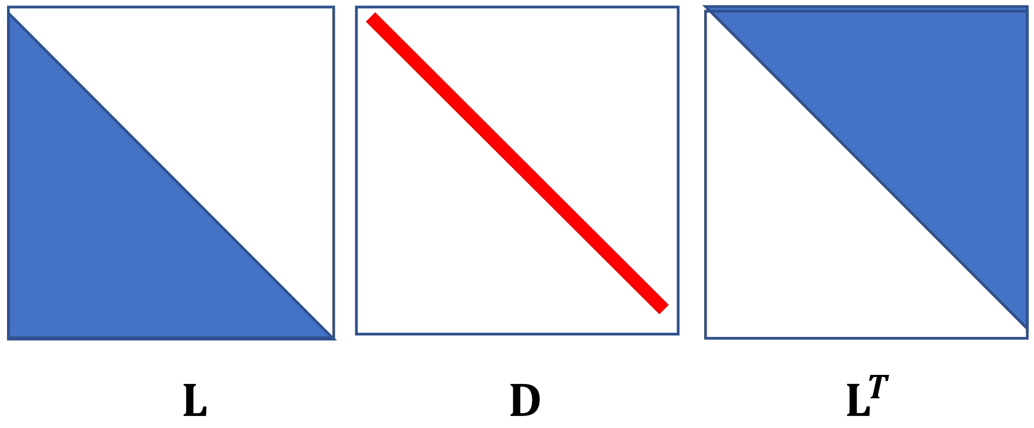
- Primal-Dual Interior-Point solver
- Search Direction with Mehrota predictor-corrector step and NT scaling
- LDL Factorization with Approximate Minimum Degree Permutation [Amestoy et al., Siam J.'96]



Non-zeros: 1.69% (4304/254016)

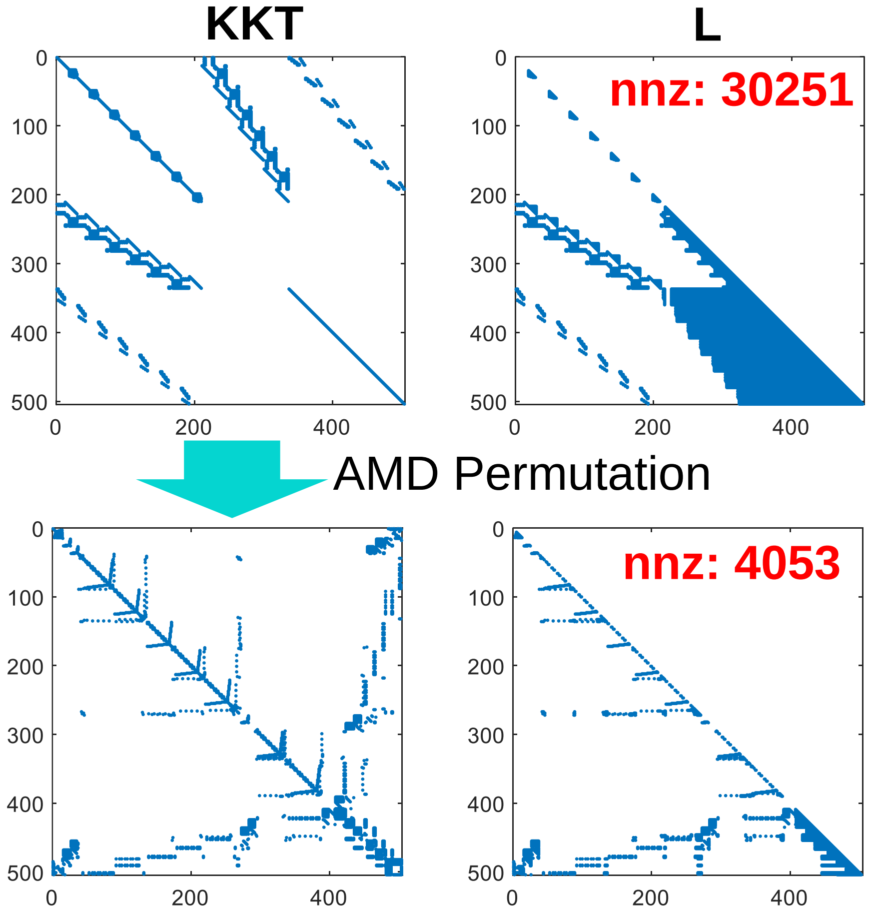
Sparse KKT Matrix

→



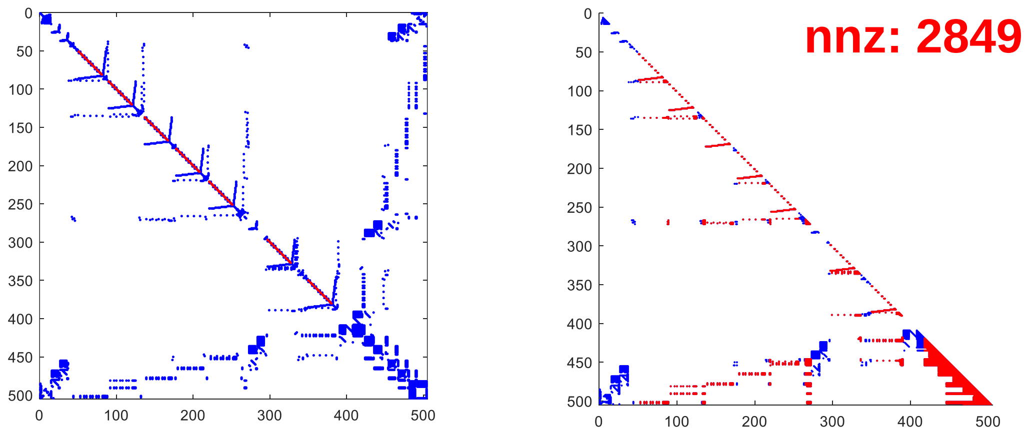
Sparse QP Solver for MPC [RA-L'19]

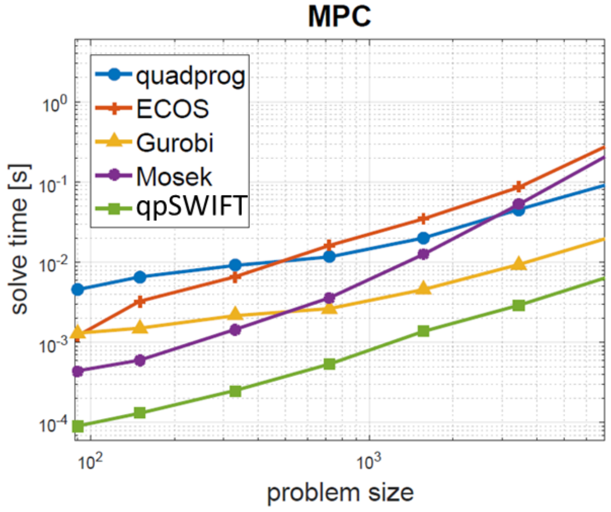
- Caching the Cholesky factor pattern

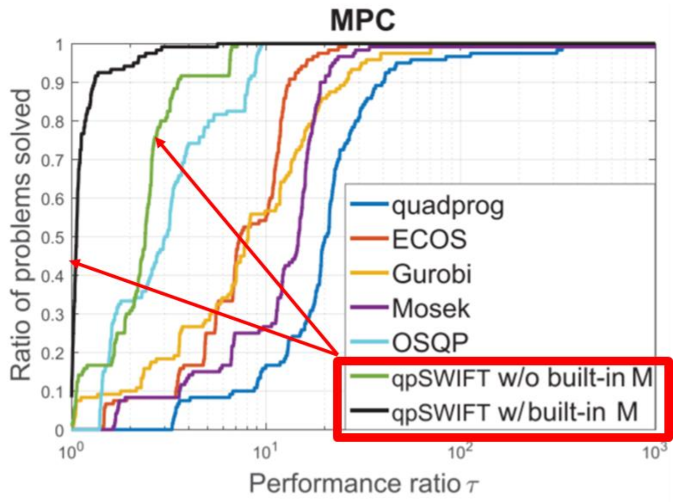


- Factorizing only rows that changes

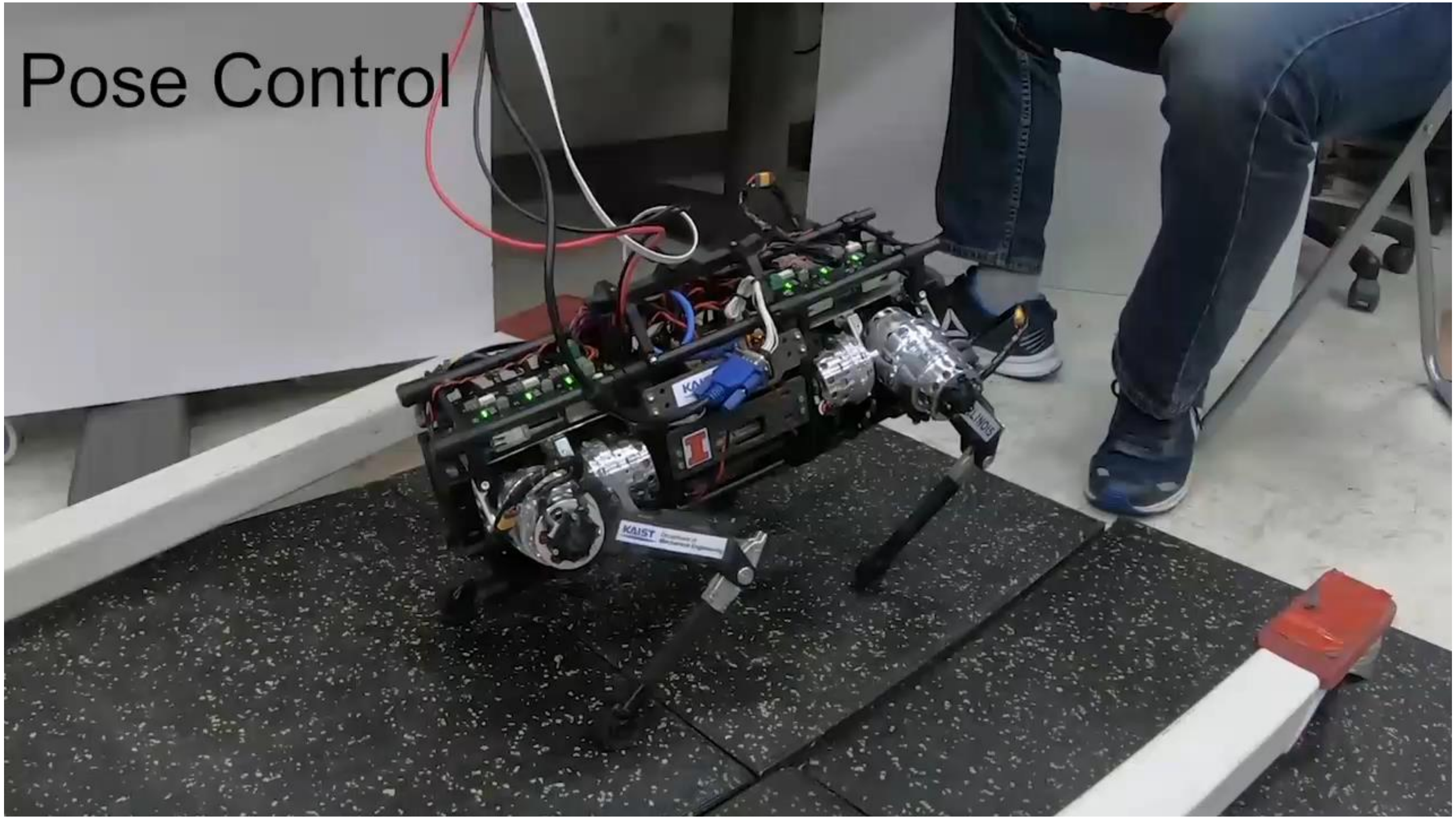
→ Avoid redundant computation





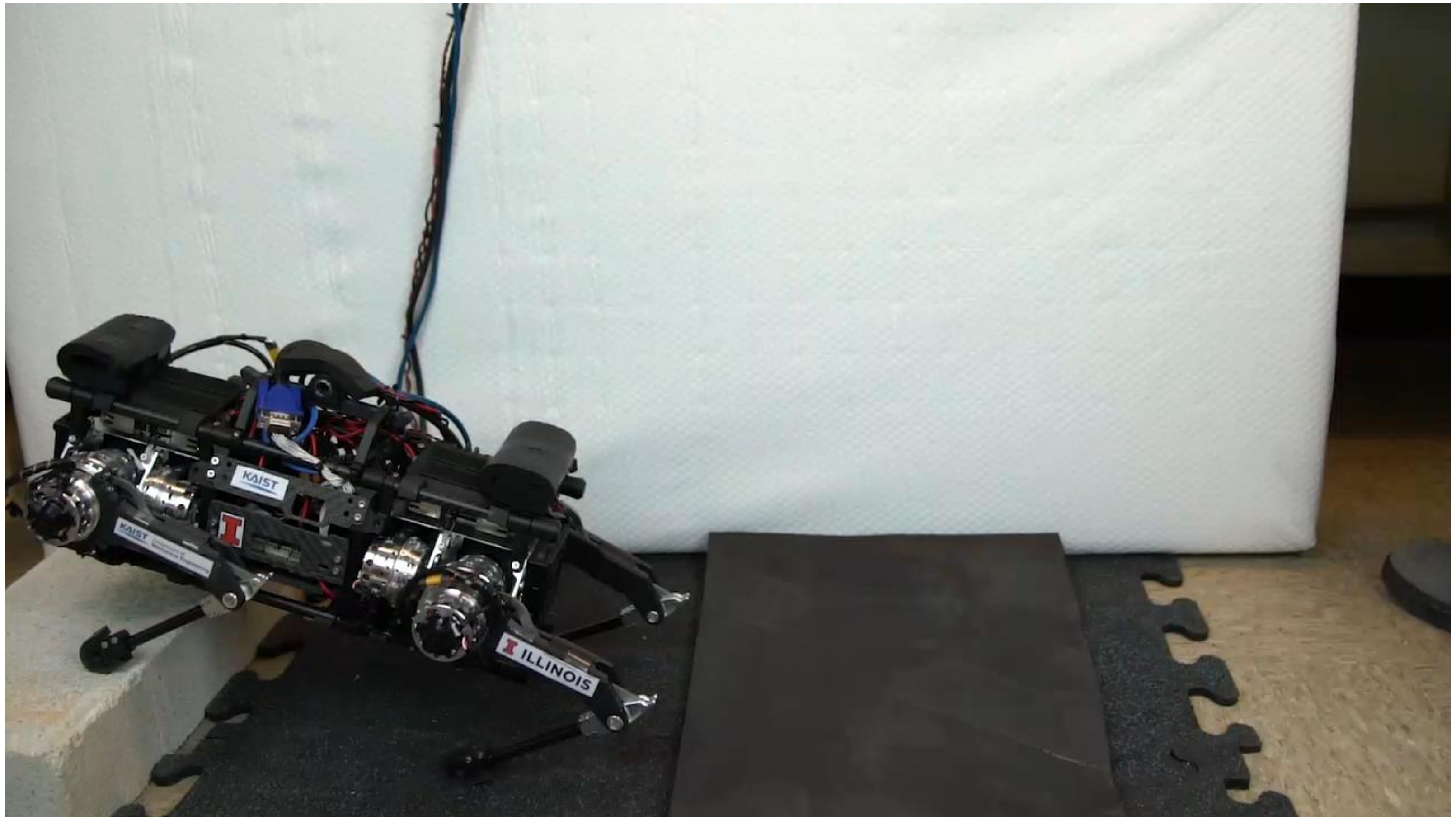


Model Predictive Control Experiments



Backflipping Experiments

- 180° backflipping controlled with RF-MPC.
- Controlled trajectory passes through the singular position of Euler angles.



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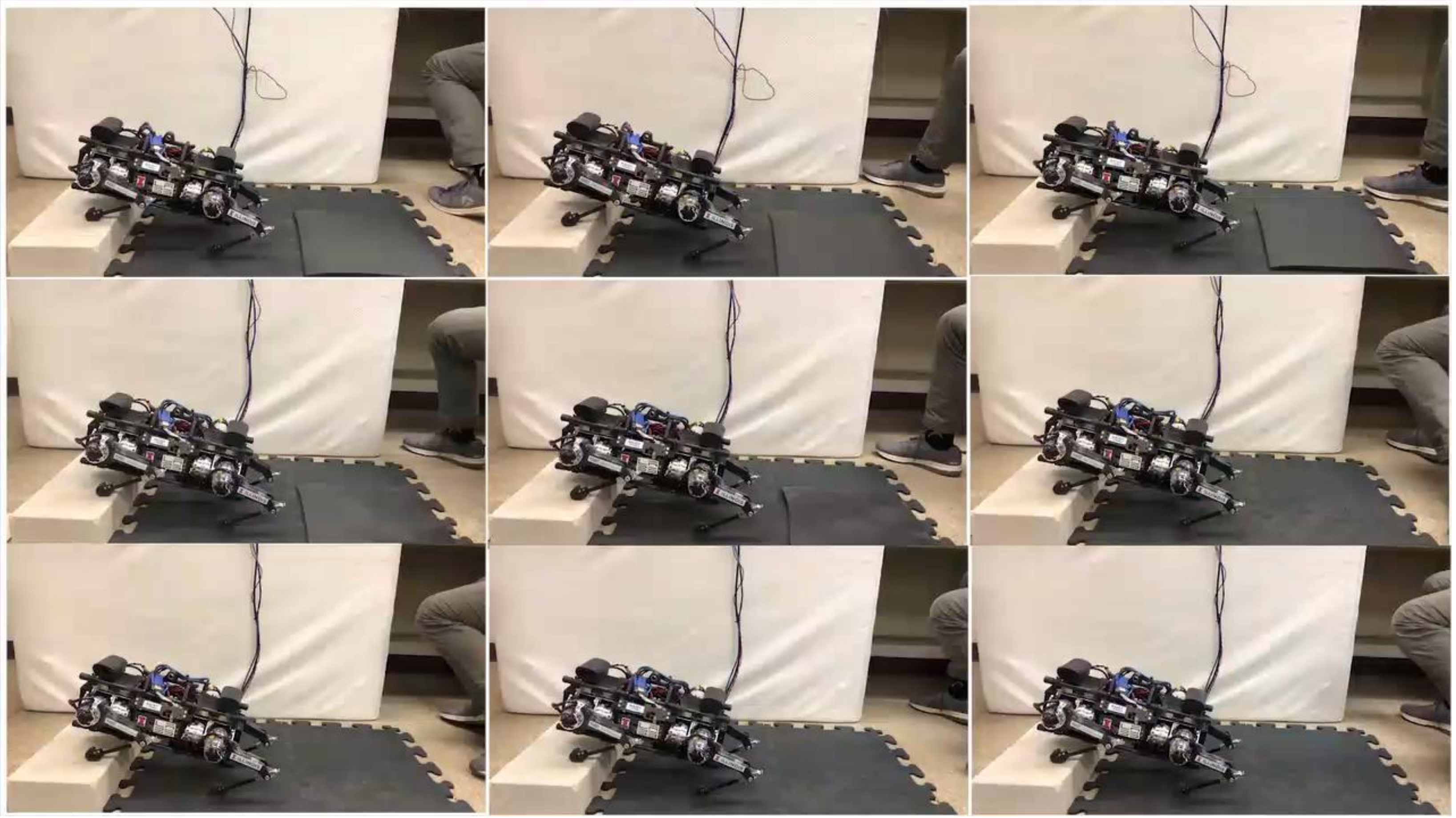


Nonlinear Representation-free MPC on SO(3)

[IROS'20, Best RoboCup Paper]

- Formulate MPC problem into optimization on SO(3) manifold
- The exponential map is selected as the retraction on a manifold.

Optimization on SO(3) manifold

$$\min_{\mathbf{R}\in SO(3)} J(\mathbf{R})$$

$$\mathbf{R}_k \in SO(3)$$
$$\mathbf{w}_k \in \mathbb{R}^3$$
$$\mathbf{p}_k \in \mathbb{R}^3$$
$$\mathbf{v}_k \in \mathbb{R}^3$$
$$\mathbf{u}_k \in \mathbb{R}^3$$

Reparameterization using retraction map

$$\mathcal{R} : SO(3) \rightarrow \mathbb{R}^3$$

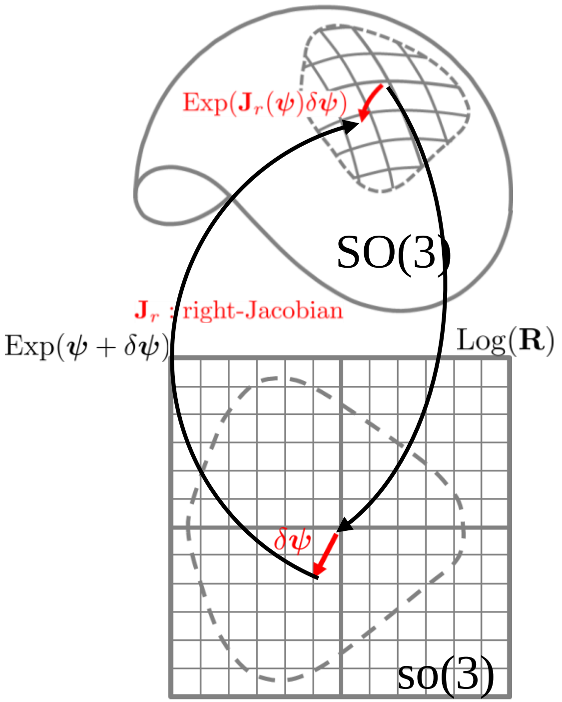
Optimization on vector space

$$\min_{\delta\boldsymbol{\psi}\in\mathbb{R}^3} J(\mathcal{R}(\delta\boldsymbol{\psi}))$$

$$\mathbf{R}_k = \overline{\mathbf{R}}_k \mathrm{Exp}(\delta\boldsymbol{\varphi}_k)$$
$$\mathbf{w}_k = \overline{\mathbf{w}}_k + \delta\mathbf{w}_k$$
$$\mathbf{p}_k = \overline{\mathbf{p}}_k + \delta\mathbf{p}_k$$
$$\mathbf{v}_k = \overline{\mathbf{v}}_k + \delta\mathbf{v}_k$$
$$\mathbf{u}_k = \overline{\mathbf{u}}_k + \delta\mathbf{u}_k$$

$$\mathbf{x}_k := [\mathbf{R}_k, \mathbf{w}_k, \mathbf{p}_k, \mathbf{v}_k]$$
$$\mathbf{u}_k := \left[\mathbf{f}_{1_k}^T, \cdots, \mathbf{f}_{c_k}^T\right]^T \in \mathbb{R}^{3c}$$

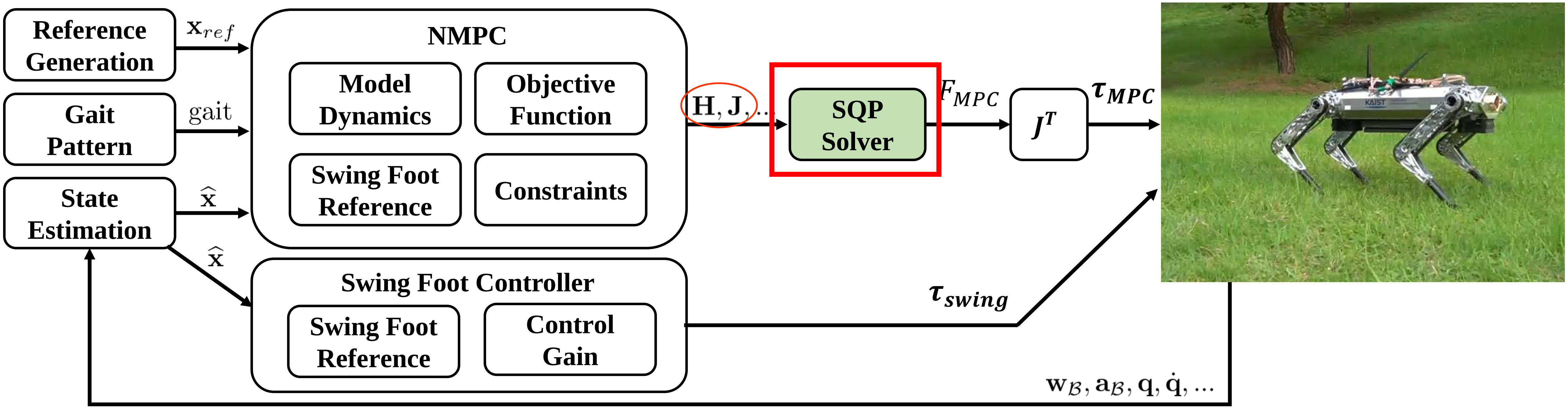
$$\delta\mathbf{x}_k := [\delta\boldsymbol{\varphi}_k^T, \delta\mathbf{w}_k^T, \delta\mathbf{p}_k^T, \delta\mathbf{v}_k^T]^T \in \mathbb{R}^{12}$$
$$\delta\mathbf{u}_k := \left[\delta\mathbf{f}_{1_k}^T, \cdots, \delta\mathbf{f}_{c_k}^T\right]^T \in \mathbb{R}^{3c}$$



[Forster et al., T-RO'16]

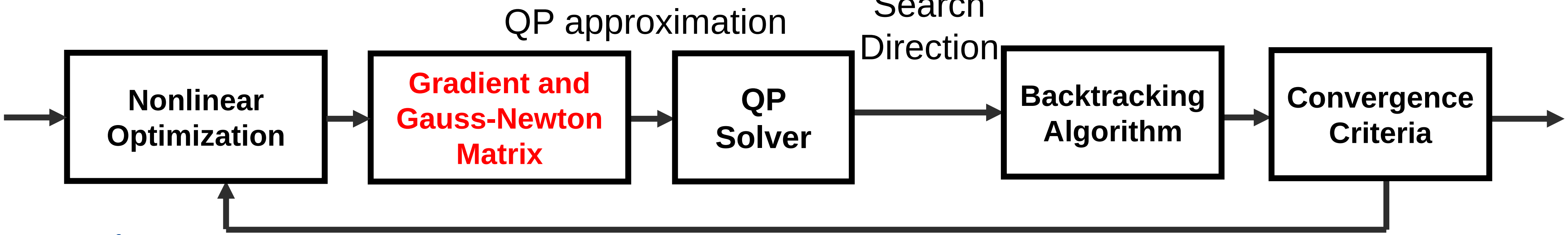
$\mathbf{H}$: Gauss-Newton Hessian matrix
$\mathbf{J}$: gradient

Nonlinear Representation-free MPC on SO(3) [IROS'20, Best RoboCup Paper]



SQP Solver

Initial guess with previous step's solution

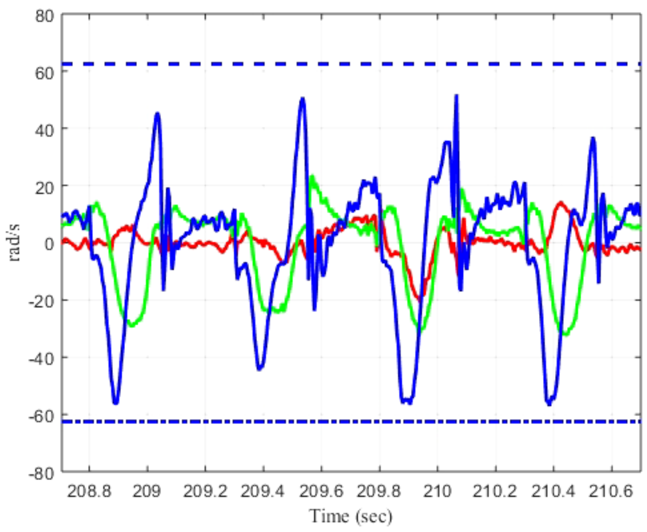


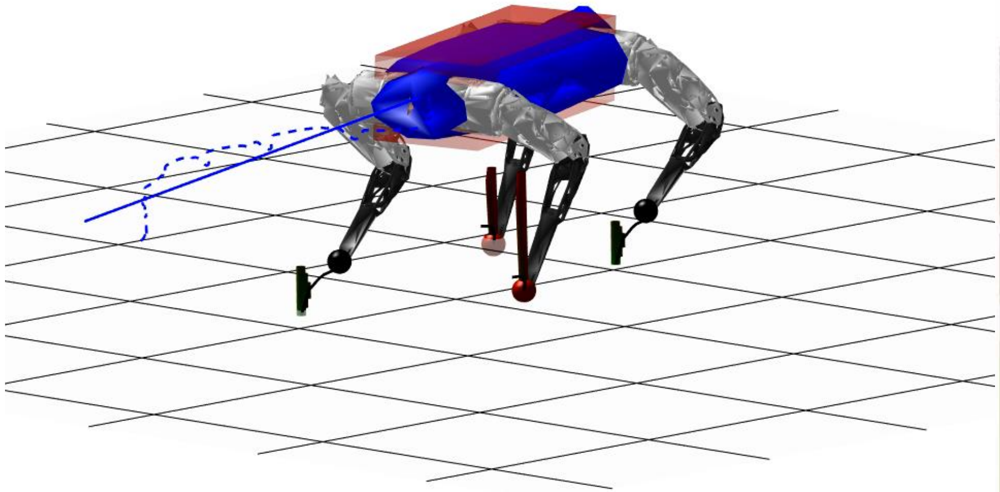
KAIST
Department of Mechanical Engineering

Experimental Results

- Push Disturbance
- Slope (40%)
- 2.9 m/sec Flying Trot

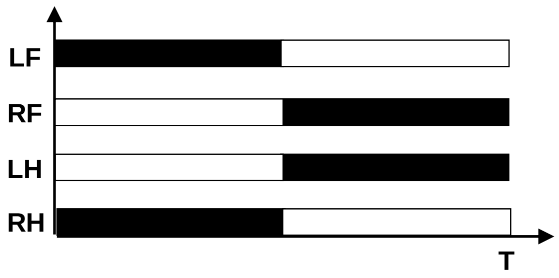
Experimental Results



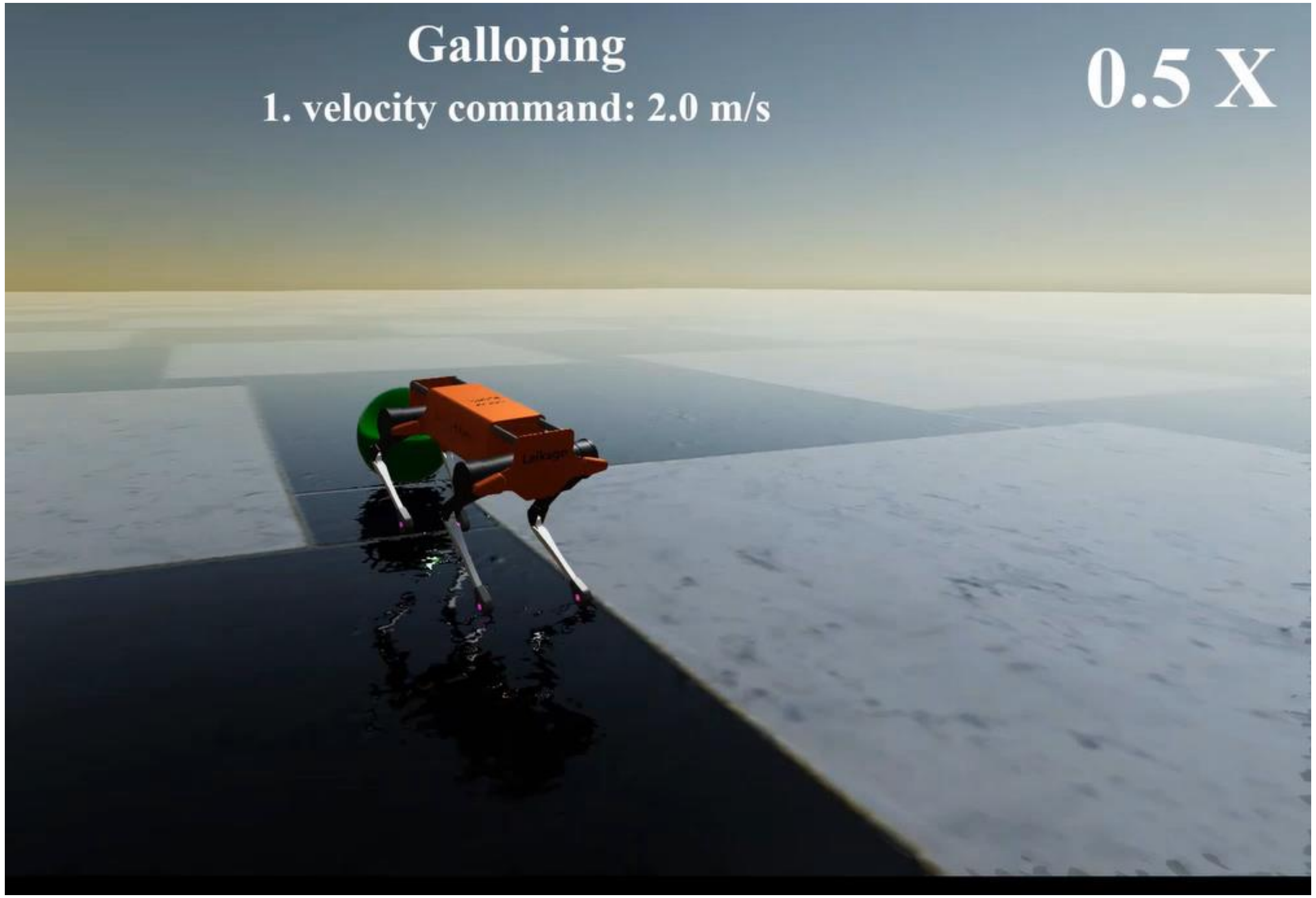


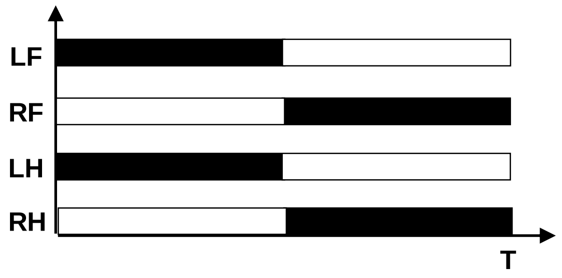
One Controller for Multiple Gaits

Galloping



Trotting

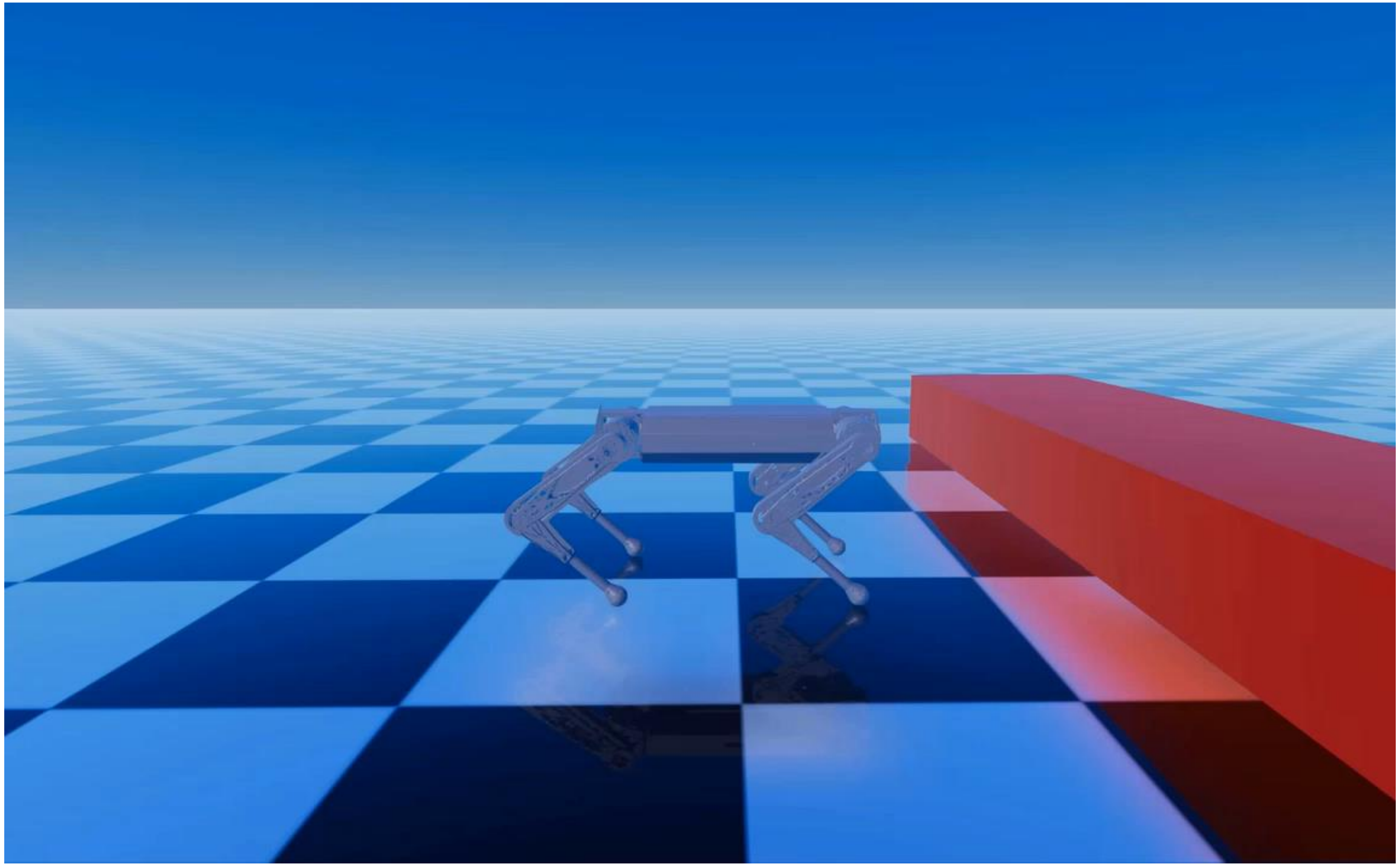




Pacing

Bounding

Gait Pattern and Motion from Motion Planner

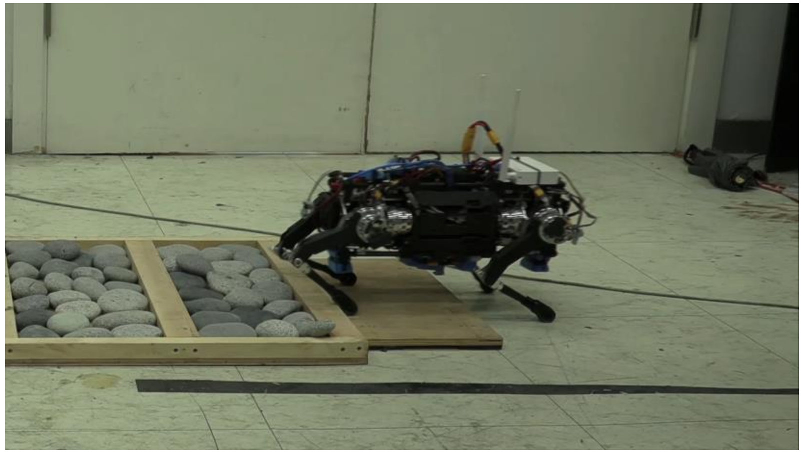


- Motions from TOWR
  [Winkler et al., RA-L’18]
- Tracking with our NMPC
- Simulation in RAISIM
  [Hwangbo et al., RA-L’19]

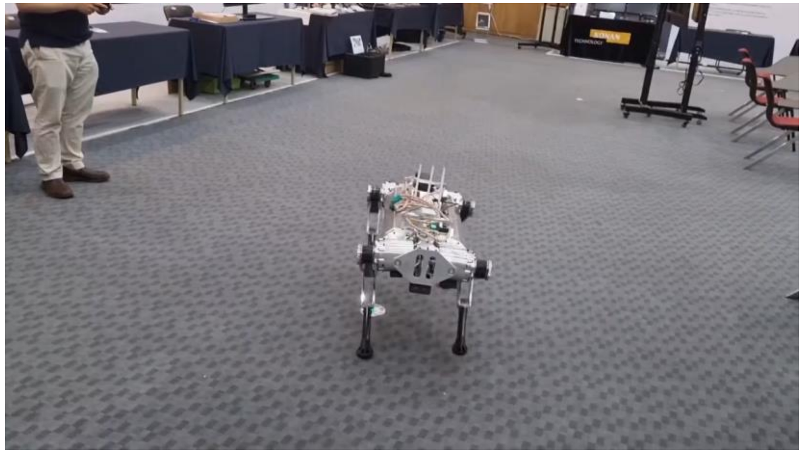
One Controller across Multiple Hardware Platforms

- With slight change of cost functions, the RF-NMPC was able to control many robots with different size scales and different actuation schemes

Electric Actuator | **Hydraulic Actuator**



BAMBY (5.5kg)



HOUND (45kg)

Hydraulic Quadruped (38kg)

Small | **Large and Heavy**

Defying Gravity: Locomotion on Ferromagnetic Surface

- With the change of GRF constraints, the NMPC is able to control vertical wall climbing locomotion (singular pose!)

Summary

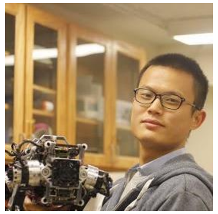
- Model predictive control could be a good controller candidate for legged robots.

  → Handle high-degrees of freedom model and constraints
  → Exploit diverse model structures and control inputs
  → Control a variety of robots, motions and gaits
  → Sparse QP solver renders real-time computation and implementation of MPC.

- Linear and nonlinear MPC can be formulated in a representation-free manner which is free from issues of Euler angles and quaternions

  →Open possibilities for controlling extreme dynamic 3D motions

- Torque control actuator design enables effective implementations of MPC on legged robots.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